

# Control of Vehicle with Power Wheeled Steerings Using Feedforward Dynamics Compensation

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## Abstract

In this paper, a construction of an efficient vehicle control method for an autonomous vehicle using the power wheeled steering (PWS) method by two driving wheels is described. First, the vehicle dynamics using the PWS method is analyzed. Next, an efficient controller for the PWS method is proposed. This controller includes a feedforward compensator which has the inverse dynamics of the vehicle and it controls both sides of the wheel angular velocities. Additionally, in order to confirm the effectiveness of the proposed control method, numerical simulation and experimental results are shown.

## 1 Introduction

The self-contained autonomous mobile robots are very interesting subject both in scientific research and the practical applications. The mobility of such robots are usually realized by the PWS mechanism, which type of vehicles has two driving wheels. Each wheel is attached at both sides of the vehicle, and the angular velocities of two wheels are independently controlled. This mechanism is relatively simple. The vehicle can rotate at the same point and easy to turn small around using this method. So, the PWS method is suitable to realize a small and light self-contained autonomous vehicle.

In order to make the vehicle tracking along the given path such as the sequence of straight lines, curves or points, the trajectory controller must be designed carefully, because the two wheels must be controlled cooperatively. Since the both side of wheels are coupled dynamically by the PWS method, the dynamics of the vehicle must be analyzed, to design a suitable controller for the PWS method.

In this paper, a construction of an efficient vehicle control method for an autonomous vehicle by the PWS mechanism using two driving wheels is described. First, the vehicle dynamics using the PWS method is analyzed. Next, an efficient controller for the PWS method is proposed. This PWS controller includes a feedforward compensator which has the inverse vehicle dynamics and it controls both sides of the wheel angular velocities. Additionally, in order to confirm the effectiveness of the proposed control method, simulation and experimental results are shown.

## 2 Vehicle control for wheeled mobile robots

As the vehicle control for an autonomous mobile vehicle, it is the reasonable way to give a path and to make the vehicle tracking

along it. This method is especially convenient for the vehicle navigation. The path can be represented by the sequence of the straight lines and curves or the sequence of the points.

Let's consider the situation of the tracking a path for an autonomous vehicle (figure 1). In order to realize such path tracking control, it is suitable to construct a vehicle control system shown in figure 2[1]. In the figure 2, the block, whose inputs are the motor torque  $(\tau_r, \tau_l)$ , and whose outputs are the wheel angular velocities  $(\omega_r, \omega_l)$ , is regarded as the **vehicle dynamics**. The block, whose inputs are  $\omega_r, \omega_l$ , and whose outputs are the real vehicle velocity  $v$  and angular velocity  $\omega$ , is regarded as the **PWS kinematics** which is represented by the matrix as following,

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \frac{R_r}{2} & \frac{R_l}{2} \\ \frac{R_r}{r} & -\frac{R_l}{r} \end{pmatrix} \begin{pmatrix} \omega_r \\ \omega_l \end{pmatrix} \quad (1)$$

The block, whose inputs are  $v$  and  $\omega$ , and whose outputs are the position  $(x, y, \theta)$ , is regarded as the **vehicle kinematics** which is represented as following,

$$\theta(t) = \int_0^t \omega(t) dt \quad (2)$$

$$x(t) = \int_0^t v(t) \cos \theta(t) dt \quad (3)$$

$$y(t) = \int_0^t v(t) \sin \theta(t) dt \quad (4)$$

By commands, the trajectory for the vehicle to track along is given to the vehicle control system. On the trajectory tracker, the reference vehicle velocity ( $v^{ref}$ ) and the reference angular velocity ( $\omega^{ref}$ ) are calculated using the given trajectory and current state variables  $(x, y, \theta, v$  and  $\omega)$ . These reference values ( $v^{ref}, \omega^{ref}$ ) determined for the robot to track along the trajectory. These variables are given to the inverse matrix in equation (1) as following,

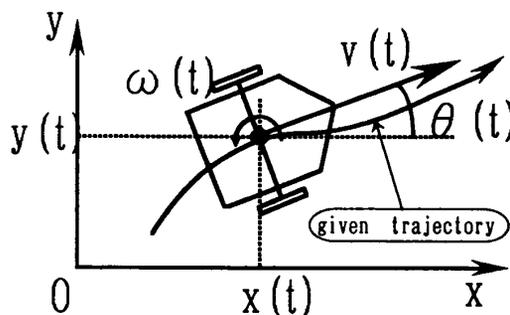


Fig. 1 A vehicle tracking along a given trajectory.

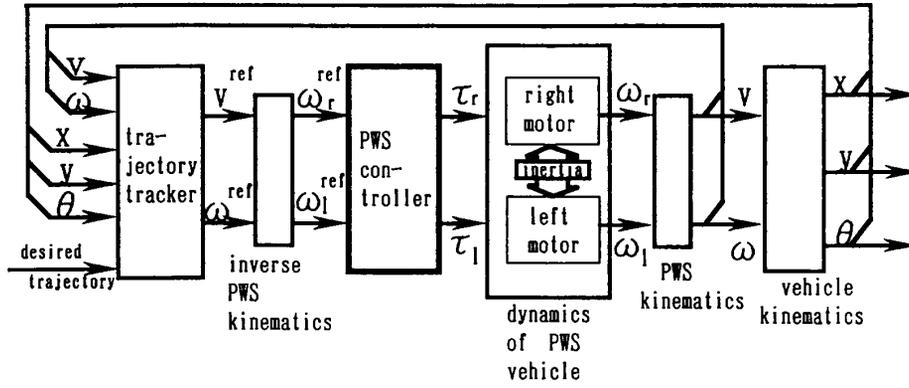


Fig. 2 Structure of a vehicle control system.

$$\begin{pmatrix} \omega_r^{ref} \\ \omega_l^{ref} \end{pmatrix} = \begin{pmatrix} \frac{1}{R_r} & \frac{T}{2R_l} \\ \frac{1}{R_l} & -\frac{T}{2R_r} \end{pmatrix} \begin{pmatrix} v^{ref} \\ \omega^{ref} \end{pmatrix} \quad (5)$$

Equation (5) may be thought as an inverse PWS kinematics.

Both sides of the motor torque ( $\tau_r, \tau_l$ ), which are given to the vehicle dynamics, are generated by the PWS controller using  $\omega_r^{ref}$  and  $\omega_l^{ref}$ . This part may be regarded as an inverse dynamics of PWS vehicle.

On the trajectory tracker, three variables ( $x, y, \theta$ ) are given and two variables ( $v^{ref}, \omega^{ref}$ ) are calculated as the inputs to the target system for tracking the trajectory. This part may be regarded as a sort of the inverse vehicle kinematics. Principally, these vehicles have two control variables ( $v, \omega$ ), and three variables ( $x, y, \theta$ ).

This type of the control system is called a nonholonomic system, and it's control is complicated. In order to control such system, it is desired to have no interactions between  $v$  and  $\omega$  in the target system. From this point of view, the PWS controller is expected to be able to cancel the dynamics of the vehicle.

When the vehicle dynamics can be perfectly known, and when the PWS controller has just inverse vehicle dynamics, the real robot velocity ( $v$ ) and the angular velocity ( $\omega$ ) can be agreed with the each reference value ( $v^{ref}$ ) and ( $\omega^{ref}$ ). In this case, the trajectory tracker will be designed easily in disregard of the vehicle dynamics.

### 3 Construction of PWS controller

#### 3.1 Analysis of the PWS vehicle dynamics

It is better to control the vehicle velocity ( $v$ ) and the vehicle angular velocity ( $\omega$ ), in order to make the vehicle tracking along the given trajectory[1].

The PWS vehicle is driven by two driving wheels which are attached at both sides, and the center of the driving wheels is regarded as the gravity center. Each parameters are defined as table 1.

Figure 3 shows the motion of the vehicle. Figure 4 illustrates the transfer relation of the torque from each motor to each wheel on the vehicle driving system.

The vehicle motion equations are,

$$M \frac{dv}{dt} = f_r + f_l \quad (6)$$

$$J \frac{d\omega}{dt} = T(f_r - f_l). \quad (7)$$

The relation between the vehicle velocity ( $v, \omega$ ) at the gravity center and the each wheel velocities ( $\omega_r, \omega_l$ ) are formulated by the equation (1).

Here,

$$v_r = R_r \omega_r, \quad v_l = R_l \omega_l. \quad (8)$$

Then,

$$f_r = \frac{1}{2} \left[ R_r \left( \frac{M}{2} + \frac{J}{T^2} \right) \frac{d\omega_r}{dt} + R_l \left( \frac{M}{2} - \frac{J}{T^2} \right) \frac{d\omega_l}{dt} \right] \quad (9)$$

$$f_l = \frac{1}{2} \left[ R_r \left( \frac{M}{2} - \frac{J}{T^2} \right) \frac{d\omega_r}{dt} + R_l \left( \frac{M}{2} + \frac{J}{T^2} \right) \frac{d\omega_l}{dt} \right]. \quad (10)$$

On the other hand, by figure 4, the transforms of the torque at the each side of the driving wheel are as follow[2],

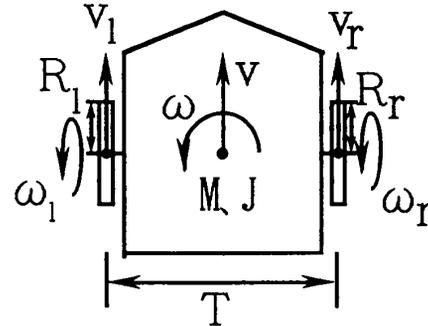


Fig. 3 Parameter of vehicle motion.

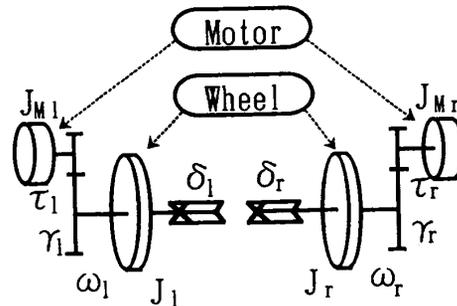


Fig. 4 Parameter of a vehicle driving system by PWS method.

$$\tau_r = J_{Mr} \frac{d\omega_{Mr}}{dt} + \frac{1}{\gamma_r} \left\{ J_r \frac{d\omega_r}{dt} + \delta_r \omega_r + R_r f_r \right\} \quad (11)$$

$$\tau_l = J_{Ml} \frac{d\omega_{Ml}}{dt} + \frac{1}{\gamma_l} \left\{ J_l \frac{d\omega_l}{dt} + \delta_l \omega_l + R_l f_l \right\}. \quad (12)$$

Accordingly, following equations can be formulated using equations (9)-(12).

$$\tau_r = A \frac{d\omega_r}{dt} + C \frac{d\omega_l}{dt} + E\omega_r, \quad (13)$$

$$\tau_l = B \frac{d\omega_l}{dt} + D \frac{d\omega_r}{dt} + F\omega_l \quad (14)$$

Where,

$$A = \frac{1}{\gamma_r} \left\{ \gamma_r^2 J_{Mr} + J_r + \frac{R_r^2}{2} \left( \frac{M}{2} + \frac{J}{T^2} \right) \right\}$$

$$B = \frac{1}{\gamma_l} \left\{ \gamma_l^2 J_{Ml} + J_l + \frac{R_l^2}{2} \left( \frac{M}{2} + \frac{J}{T^2} \right) \right\}$$

$$C = \frac{1}{\gamma_r} \left\{ \frac{R_r R_l}{2} \left( \frac{M}{2} - \frac{J}{T^2} \right) \right\}$$

$$D = \frac{1}{\gamma_l} \left\{ \frac{R_r R_l}{2} \left( \frac{M}{2} - \frac{J}{T^2} \right) \right\}$$

$$E = \frac{1}{\gamma_r} \delta_r$$

$$F = \frac{1}{\gamma_l} \delta_l.$$

The equations (13) and (14) describe the vehicle dynamics based on the PWS method.

### 3.2 PWS controller using feedforward compensator

The feedforward compensator which has the inverse vehicle dynamics, is designed. Its inputs to the controller are both sides of the reference wheel angular velocity. The outputs are both sides of the reference motor torque.

When the velocities of the both wheels interact mutually by the vehicle dynamics as the previous analysis result, it is difficult to make the response of the wheel angular velocity to stabilize and track quickly by the conventional feedback control. So the vehicle dynamics must be considered, the controller unique to the PWS method must be constructed, and the response of the wheel

$M$	mass of the vehicle
$J$	inertia of the vehicle
$J_{l/r}$	inertia of the left/right wheel axis
$J_{Ml/r}$	inertia of the left/right motor axis
$\delta_{l/r}$	friction constant of the left/right wheel
$\gamma_{l/r}$	gear ratio of the left/right wheel
$\tau_{l/r}$	torque of the left/right motor
$v$	velocity of the gravity center
$v_{l/r}$	velocity of the left/right wheel
$\omega$	angular velocity of the gravity center
$\omega_{l/r}$	angular velocity of the left/right wheel
$\omega_{Ml/r}$	angular velocity of the left/right motor
$T$	tread of the vehicle
$R_{l/r}$	radius of the left/right wheel
$f_{l/r}$	driven force at the left/right wheel

Table 1. Nomenclature

angular velocity must be made stabilizing and tracking quickly to reference values. Therefore, it is easy to design the trajectory tracker, if the response of the PWS control level is excellent, as  $v$  and  $\omega$  can be controlled independently.

When the reference velocity and angular velocity of the vehicle ( $v^{ref}, \omega^{ref}$ ) are given, the reference angular velocity of the both wheels ( $\omega_r^{ref}, \omega_l^{ref}$ ) can be calculated by the inverse PWS kinematics (5). The equations (13) and (14) represents the vehicle dynamics ( or the relation between the motor torque and the angular velocity of the both wheels). The equations (13) and (14) are noted that, when the inputs are the motor torque ( $\tau_r, \tau_l$ ), the outputs are the wheel angular acceleration ( $d\omega_r/dt, d\omega_l/dt$ ) at the wheel angular velocity  $\omega_r$  and  $\omega_l$ .

Therefore, a feedforward compensator which has the inverse vehicle dynamics is constructed. The relation between the reference angular velocity to realize and the motor torque can be deduced from the equations (13) and (14). First, the differentiated values ( $(d\omega_r/dt)^{ref}, (d\omega_l/dt)^{ref}$ ) are calculated from the reference values ( $\omega_r^{ref}, \omega_l^{ref}$ ). When  $(d\omega_r/dt)^{ref}$  and  $(d\omega_l/dt)^{ref}$  are given, the torque to realize the reference wheel angular velocity can be calculated by the equations as follow,

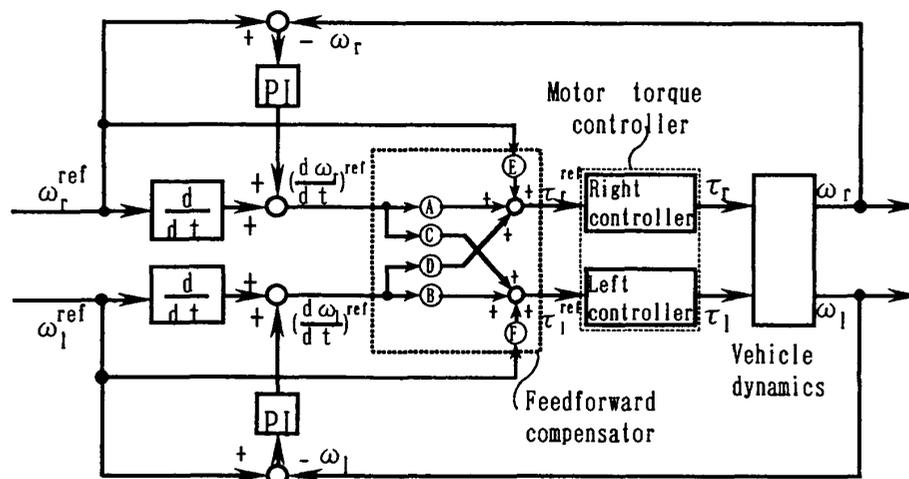


Fig. 5 Construction of a PWS controller.

$$\tau_r^{ref} = A \left( \frac{d\omega_r}{dt} \right)^{ref} + C \left( \frac{d\omega_l}{dt} \right)^{ref} + E \omega_r^{ref} \quad (15)$$

$$\tau_l^{ref} = B \left( \frac{d\omega_l}{dt} \right)^{ref} + D \left( \frac{d\omega_r}{dt} \right)^{ref} + F \omega_l^{ref}. \quad (16)$$

Where each parameters(A..F) are defined previously. The equations (15) and (16) give the feedforward compensator which can cancel the vehicle dynamics by the equations (13) and (14). Then, the derivative outputs by the vehicle dynamics  $(d\omega_r/dt, d\omega_l/dt)$  agree with the inputs to the feedforward compensator  $((d\omega_r/dt)^{ref}, (d\omega_l/dt)^{ref})$ .

However, the parameter error between the real vehicle and the model will exist because of the analysis or the noise in the dynamics. In order to absorb these error or noise, we must use the conventional PI feedback loop for the each wheel. For this feedback loop, the high gain controller is not required.

Figure 5 shows the proposed PWS controller using a feedforward compensator.

In the case of the DC servo motor, the under equation on the torque and the motor current is applicable.

$$\tau_{l/r} = \kappa_{l/r} i_{l/r}$$

Here,

$\kappa_{l/r}$  are the torque constant of left/right motors, and  $i_{l/r}$  are the left/right motor current.

So, the torque control is realized by the current control. An effective current control method has been proposed[3], when the DC-servomotor is driven by the PWM switching method and the software servo technique.

## 4 Experiments

### 4.1 Experimental system 'Yamabico' mobile robot

The authors are developing series of self-contained autonomous robot 'Yamabico' (figure 6). Table 2 shows the parameters of 'Yamabico'. 'Yamabico' is a typical autonomous vehicle by the PWS method. Each parameter is measured or estimated.  $M, \gamma_r, \gamma_l, T, R_r$  and  $R_l$  are measured.  $J$  is calculated as the vehicle is regarded as a rectangular prism.  $J_r, J_l, J_{Mr}$  and  $J_{Ml}$  are calculated as the wheel or the gears are regarded as circular plate.  $\delta_r$  and  $\delta_l$  are estimated by the experiment. In the experiment, the stabilized vehicle velocity is measured, when the constant torque is given to the motor.

As an example, the values(A..F) of 'Yamabico' are approximately calculated as following,

$$A = B = 9.347 \text{ [Nmsec}^2\text{]}$$

$$C = D = 0.432 \text{ [Nmsec}^2\text{]}$$

$$E = F = 9.565 \text{ [Nmsec].}$$

The Pulse width modulation (PWM) switching method is adopted to control the DC servo motors. The vehicle controller is realized by a single computer board using a 68000CPU with 128KBite RAM and 64KBite ROM for the local memory, a programmable timer module (PTM) for generating sampling signal, serial interface (ACIA), PWM signal generator, 1Kbite dual port memory (DPM) for communicating with the master module and 2 floor board interface for extending. Figure 7 shows the hardware structure of the vehicle controller.

Its control program is written by the C-langage, and the sampling period is 5 msec.

$M$	13.0	[Kg]
$J$	0.2	[Kgm <sup>2</sup> ]
$J_r$	0.002	[Kgm <sup>2</sup> ]
$J_l$	0.002	[Kgm <sup>2</sup> ]
$J_{Mr}$	0.000001	[Kgm <sup>2</sup> ]
$J_{Ml}$	0.000001	[Kgm <sup>2</sup> ]
$\delta_r$	0.020	[Kgm <sup>2</sup> /sec]
$\delta_l$	0.020	[Kgm <sup>2</sup> /sec]
$\gamma_r$	39.5	
$\gamma_l$	39.5	
$T$	0.336	[m]
$R_r$	0.062	[m]
$R_l$	0.062	[m]

Table 2. Parameters of 'Yamabico'.



Fig. 6 A self-contained autonomous mobile robot 'Yamabico'.

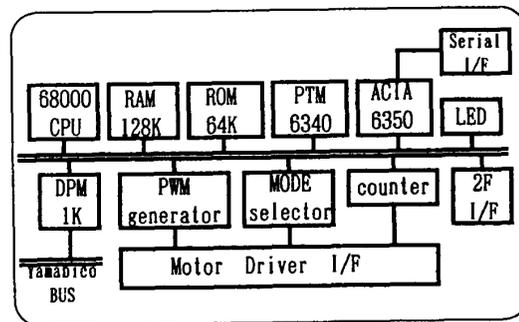


Fig. 7 Hardware structure of vehicle control module.

## 4.2 Experimental results

In order to evaluate the effectiveness of the proposed feedforward control method in the previous section, the proposed control method was simulated on the computer and we had some experiments by the Yamabico mobile robot.

The vehicle is supposed to move on a sort of the given curve as shown in figure 8. Where, each parameter for the controller and vehicle dynamics are used for 'Yamabico' in table 2.

Figure 9 shows the simulated responses of  $v_{l/r}$ ,  $v_{l/r}^{ref}$  and  $\tau_{r/l}$ , using the traditional feedback control and using the proposed feedforward compensation in the PWS controller. In the case of including about 10% error between the controller parameters and the vehicle dynamics, the result is recorded in figure 9 (a) and (b).

Figure 10 shows the experimental responses of Yamabico using the traditional feedback control and using the proposed feedforward compensation in the PWS controller. The result is recorded in figure 10 (a) and (b).

These results show that the responses using the feedforward controller is improved. Therefore, the availability of the proposed control method using the feedforward compensator is confirmed.

## 5 Conclusion

In this paper, a construction of an efficient vehicle control method for autonomous mobile robots using the power wheeled steering (PWS) method by two driving wheels was described. First, the vehicle dynamics using the PWS method was analyzed. Next, an efficient controller structure for the PWS method was described. This controller includes a feedforward compensator which has the inverse dynamics of the vehicle and it controls both sides of the wheel angular velocities. Additionally, in order to confirm the effectiveness of the proposed control method, numerical simulation results and experimental results were shown.

## References

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- [2] Y.Sasaki: "System design of digital servo", Kindai-tosho, pp.104-119, 1989 (in Japanese).
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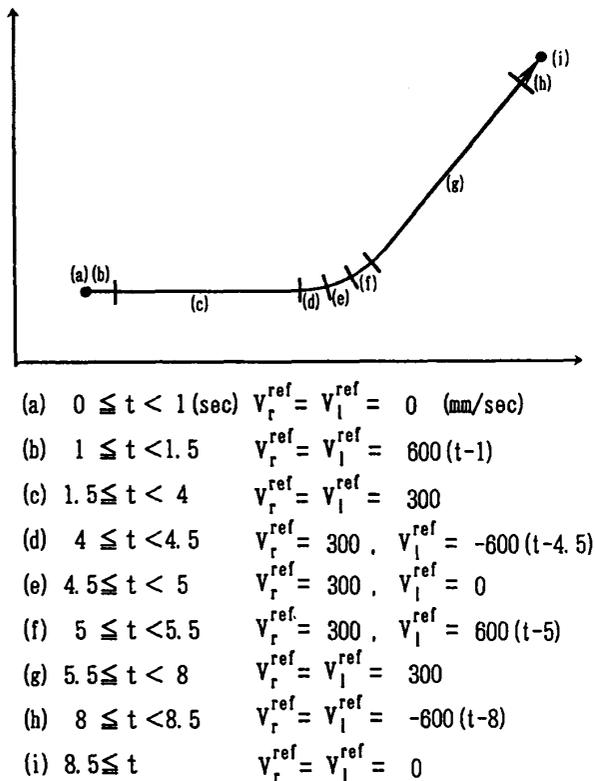
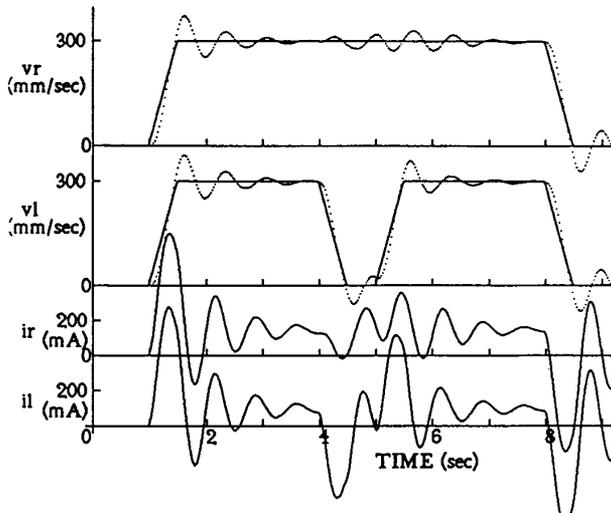


Fig. 8 Simulated and experimental vehicle motion.

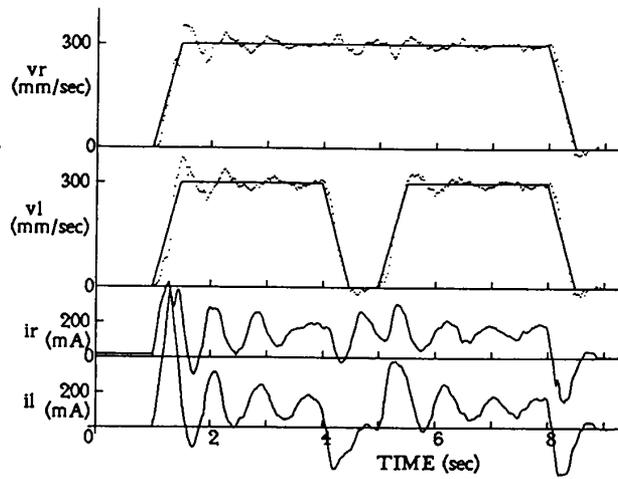


$$i_{r/l} = \frac{1}{K_{r/l}} \tau_{r/l} \quad (K_{r/l}: \text{right/left motor torque constant})$$

$$v_{r/l} = R_{r/l} \omega_{r/l} \quad (R_{r/l}: \text{right/left wheel radius})$$

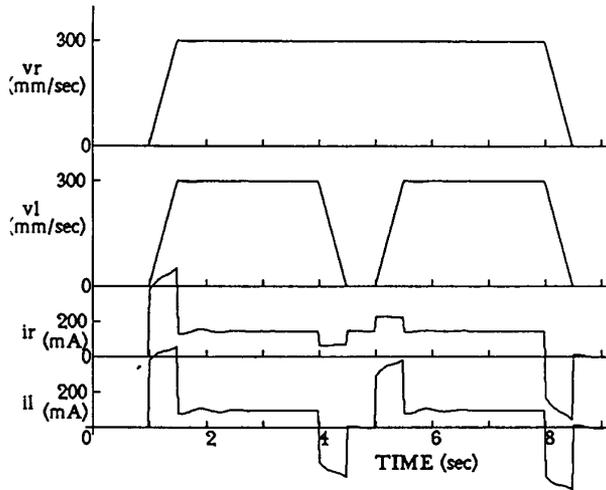
real line: reference value  
dotted line: real value

(a) using PI controller



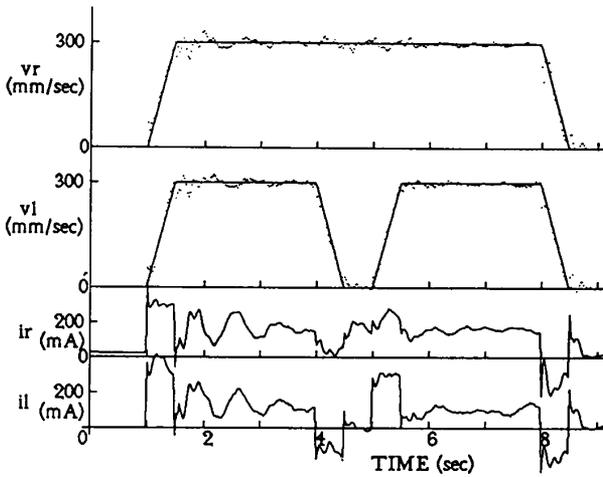
real line: reference value  
dotted line: real value

(a) using PI controller



real line: reference value  
dotted line: real value

(b) using feedforward compensator



real line: reference value  
dotted line: real value

(b) using feedforward compensator

Fig. 9 Numerical simulation results of proposed PWS control method.

Fig. 10 Experimental results of proposed PWS control method.