Then, using similar arguments to the ones in [3], it can be shown that [3, Th. 1] holds.

Moreover, [3, Th. 2] holds with the following minor changes: $V_k^{(p)}(x^{(p)}, M)$ replaces $V_k^{(p)})(x^{(p)}, \overline{x}^{(p)})$ in statement 1. Statement 2 is unchanged. Statement 3 is modified to the following explicit formula for the Gittins index:

$$\gamma_N^{(p)}\left(x^{(p)}\right) = \max_{\lambda_{i,N} \in \Lambda_N} \frac{\overline{M}\lambda'_{i,N}(3)\, x^{(p)}}{(\lambda_{i,N}(3) - \lambda_{i,n}(1))'\, x^{(p)} + \overline{M}} \tag{9}$$

where each vector $\lambda_{i,N} \in \Lambda_N^{(p)}$ is of the form

$$\lambda_{i,k} = [\,\lambda'_{i,k}(1) \quad 0 \quad \lambda'_{i,k}(3) \quad 0]' \quad \text{where } \lambda_{i,k}(1),\, \lambda_{i,k}(3) \in \mathbb{R}^{\mathcal{N}_p}. \tag{10}$$

Statement 3 above gives an explicit formula for the Gittins index of the HMM multi-armed bandit problem. Recall that $x_k^{(p)}$ is the information state computed by the pth HMM filter at time k. Given that we can compute set of vectors $\Lambda_N^{(p)}$, (9) gives an explicit expression for the Gittins index $\gamma_N^{(p)}(x_k^{(p)})$ at any time k for project p. Note if all elements of $R(p)$ are identical, then $\gamma^{(p)}(x) = \overline{M}$ for all x. Sections II-E, III, and IV of [3], including the beam scheduling algorithm for a hybrid sensor of Section III-C, still hold.

It is worthwhile noting that the above solution is computationally simpler as the information state π is a $2(\mathcal{N}_p + 1)$-dimensional vector, whereas the information state π considered in [3] is a $\mathcal{N}_p^2$-dimensional vector.

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Beamforming Using the Fractional Fourier Transform

İmam Şamil Yetik and Arye Nehorai

***Abstract*—We present a new method of beamforming using the fractional Fourier transform (FrFT). This method encompasses the conventional minimum mean-squared error (MSE) beamforming in the frequency domain or spatial domain as special cases. It is especially useful for applications involving chirp signals such as signal enhancement problems with accelerating sinusoidal sources where the Doppler effect generates chirp signals and a frequency shift and active radar problems where chirp signals are transmitted. Numerical examples demonstrate the potential advantage of the proposed method over the ordinary frequency or spatial domain beamforming for a moving source scenario.**

***Index Terms*—Beamforming, fractional fourier transform, sensor array signal processing, time-frequency analysis.**

I. INTRODUCTION

Beamforming is a widely used tool in sensor array signal processing for various goals such as: signal enhancement, interference suppression, and direction of arrival (DOA) estimation. Essentially, beamforming is a filtering of signals arriving at distributed sensors. The filtering weights at the sensors are chosen to achieve a certain goal. Spatial filtering is useful in many applications since the signals of interest and the interference are spatially separated. Generally, a beamformer also includes temporal filtering along with the spatial filtering to exploit spectral differences. It uses a weighted sum of the sensor outputs at certain time instants. In other words, beamforming is a linear combination of the temporal outputs of the multiple sensors. Mathematically, we can express a general beamformer operation as

$$y(t) = \sum_{i=1}^{J} \sum_{p=0}^{K-1} w_{i,p}^{*} x_i(t - pT) \tag{1}$$

where $y(t)$ is the beamformer output, $w_{i,p}$'s are the weights of the beamformer, $x_i(t)$'s are the signals arriving at the sensors, $K-1$ is the number of delays in each of the sensor channels, J is the number of sensors, T is the duration of a single time delay, and the superscript "*" denotes complex conjugate. We can write (1) in vector form as follows:

$$y(t) = \boldsymbol{w}^H \boldsymbol{x}(t) \tag{2}$$

where

$$\begin{aligned} \boldsymbol{x}(t) &= [x_1(t), x_1(t-T), \ldots, x_1(t-(K-1)T), \ldots \\ &\qquad x_2(t), \ldots, x_J(t-(K-1)T)]^T \\ \boldsymbol{w} &= [w_{1,0},\, w_{1,1}, \ldots, w_{1,K-1}, \ldots \\ &\qquad w_{2,0}, \ldots, w_{J,K-1}]^H \end{aligned} \tag{3}$$

where the superscript "^{T}" denotes transpose and "^{H}" Hermitian conjugate. Some beamformers deviate from this general form to meet certain needs. For example, when the signal of interest is broadband, it

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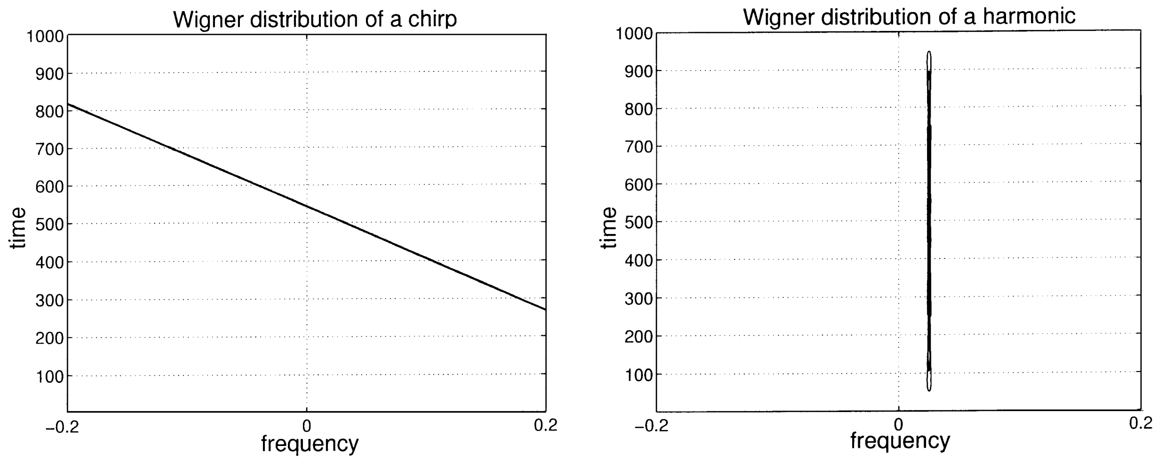

Fig. 1. Rotational effect of the FrFT in the time-frequency plane: A chirp that is an oblique line in the time-frequency plane transforms into a harmonic that is a vertical line in this plane.

is a good idea to perform beamforming in the frequency domain (i.e., following Fourier transform) rather than spatial domain.

There exists a vast number of beamforming algorithms in the literature. Here, we will briefly review only a few general approaches. Each of these has its advantages and disadvantages, depending on the application and the prior knowledge we have about the signal. Maximum signal-to-noise ratio (SNR) beamforming [1] chooses the weights so that the SNR at the output of the beamformer is maximized. The optimal weights require knowledge of the second-order statistics of the noise and signal. The multiple side-lobe canceler [2] aims to cancel the effect of auxiliary channels that are used to measure noise and interference. The Capon beamformer [3], which is a special case of the linearly constrained minimum variance beamformer [4], minimizes the output power to ensure that the effect of noise is minimized, with the constraint that the magnitude of the frequency response in a certain direction is unity. Use of a reference signal [5] suggests the design of the beamformer so that the MSE between its output and a desired signal is minimized. The method proposed in this paper can be viewed as a generalization of the last one. To find a more extensive treatment of beamforming, see [6].

The motivation behind the proposed method is the ability of the fractional Fourier transform (FrFT) to process the chirp signals better than the ordinary Fourier transform (FT). In array signal processing, chirp signals are encountered for example in problems where a sinusoidal source is accelerating or active radar problems where chirp signals are transmitted. Acceleration of the source causes its sinusoids to arrive at the sensors as chirp signals. Therefore, replacing the FT with the FrFT should improve the performance considerably.

In Section II, we give a brief overview of the FrFT and explain the advantage of using it in this specific problem. Section III explains the proposed beamforming method. Numerical examples for a moving source is given in the Section IV. Section V is devoted to a discussion of the results and future work.

II. Fractional Fourier Transform

The FrFT [7] is essentially a time-varying filter. More specifically, it is a one-parameter generalization of the FT. We compute the FrFT by using this parameter as the functional power of the ordinary FT [7]. Letting $x(u)$ be an arbitrary signal, its ath-order FrFT is defined as

$$x_a(u) = \int_{\infty}^{\infty} K_a(u,u')x(u')du' \tag{4}$$

where

$$\begin{aligned} K_a(u,u') &= A_\phi \exp\left[i\pi(\cot\phi u^2 - 2\csc\phi u u' + \cot\phi u'^2)\right] \\ \phi &= \frac{a\pi}{2} \\ A_\phi &= \sqrt{1 - i\cot\phi}. \end{aligned}$$

The square root above is defined such that the argument of the result lies in the interval $(-\pi/2, \pi/2]$. When a is an even integer, the above kernel is undefined. However, it is possible to show that as a approaches an even integer, the kernel approaches a delta function. That is, $K_{4l}(u,u') = \delta(u-u')$ and $K_{4l\pm2}(u,u') = \delta(u+u')$, where l is an arbitrary integer. The FrFT reduces to the ordinary FT for $a = 1$ and identity operation for $a = 0$. It is index additive, that is, the a_1th-order FrFT of the a_2th-order FrFT is equal to the $(a_1 + a_2)$th-order FrFT. In addition, there exists a fast implementation of the FrFT [8] with implementation cost of order $N\log(N)$, where N is the signal temporal length.

The effect of the FrFT is most easily seen in the time-frequency plane. Here, we use the Wigner distribution to illustrate the effect of the FrFT on a signal since it is one of the most popular time-frequency distributions and has many useful properties [9]. It is defined as

$$\mathcal{W}(u,v) = \int_{-\infty}^{\infty} x\left(u + \frac{u'}{2}\right) x\left(u - \frac{u'}{2}\right) \exp[i2\pi v u']\, du' \tag{5}$$

where v denotes the frequency variable, and u is the time or space variable. The Wigner distribution gives an idea of how the energy of a signal is distributed in time and frequency. The effect of the FrFT on the Wigner distribution of a signal is simply a clockwise rotation by an angle ϕ in the time-frequency plane, as illustrated for a chirp signal in Fig. 1.

The FrFT has found many applications in digital signal processing such as filtering [10], signal restoration [11], system synthesis [12], mutual intensity synthesis [13], system decomposition [14], [15], optimum Wiener filtering [16], image restoration [17], and perspective projections [18]. More complete treatment of the FrFT and many of its optics and digital signal processing applications can be found in [7].

The FrFT suggests a potential improvement in any application where the ordinary FT is used since it provides an extra degree of freedom corresponding to the choice of the order a. We can attempt to improve the solution to any problem that utilizes the FT by carrying the extra parameter throughout the solution and then optimizing over this parameter. The FrFT is most likely to improve the solutions to problems where chirp signals are involved. This is because a chirp signal forms

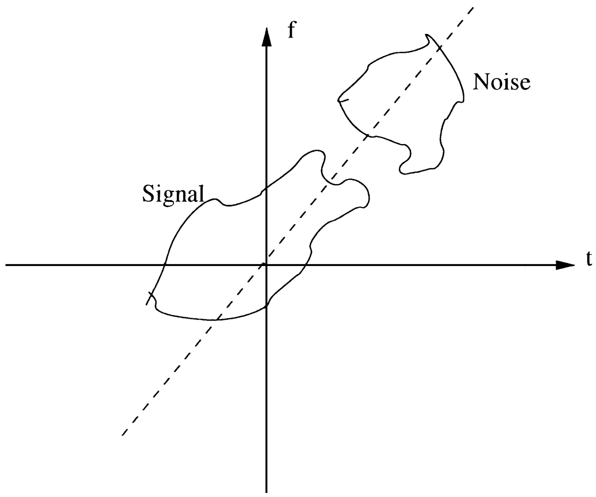

Fig. 2. Separation of signal and noise in an appropriate domain.

a line in the time-frequency plane, and therefore, there exists an order for which such a signal is compact. Chirp signals are not compact in the spatial or time domain. Thus, in many such cases, we can filter out the signal easily in an appropriate fractional Fourier domain when it is not possible to separate the signal and noise in space or frequency domain, as shown in Fig. 2. A sinusoid emitted from an accelerating source will arrive the sensors as a chirp signal, and therefore, the FrFT beamforming can improve the beamformer performance. Indeed, our computer simulations show that much smaller errors are obtained when the FrFT beamformer is used.

III. Beamforming Using the FrFT

The method we propose generalizes the minimum MSE beamformer [5]. In the latter method, the goal is to minimize the MSE between the beamformer output and the desired signal. The desired signal is determined by the problem at hand. In a moving source problem, the desired signal is the signal emitted by the source, which we want to obtain as free of noise as possible. In an active radar problem, it may be the signal reflected from the target. It may have different meanings in other beamforming applications that we do not mention here. Mathematically, the optimal weights $\boldsymbol{w}_{\rm opt}$ given by

$$\boldsymbol{w}_{\rm opt} = \min_{\boldsymbol{w}} E\{\|y(t) - y_d(t)\|^2\} \tag{6}$$

where $y_d(t)$ denotes the desired signal, $y(t)$ the beamformer output, and $\|\cdot\|$ the L_2 norm given by $\|y(t)\|^2 = \int_{-\infty}^{\infty} y(t)y^*(t)\,dt$. The optimum weights can be calculated by substituting the beamformer output (2) in the MSE expression (6) to be minimized. Solving these equations give the optimum weights

$$\boldsymbol{w}_{\rm opt} = R_x^{-1}\mathbf{r}_{xd} \tag{7}$$

where R_x is the covariance of the measurements at the sensors, and $\mathbf{r}_{xd}$ is the cross-covariance between the measurements at the sensors and the desired signal. The beamformer output is given by (2).

The above result corresponds to the spatial filtering of the signals arriving at the sensors. We extend this result by using filtering in a fractional Fourier domain rather than the spatial domain. Fig. 3 shows the general structure of the proposed beamformer. The measurements at the sensors are transformed into the ath fractional Fourier domain; then, beamforming is performed in this domain, and the output is transformed back into the time domain by using the inverse FrFT. We can summarize these operations by writing the input-output relationship explicitly

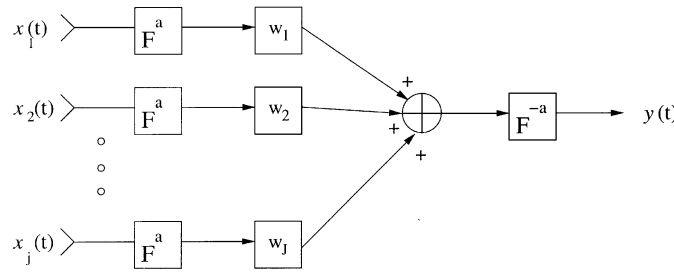

Fig. 3. Block diagram of the proposed FrFT beamformer. F^a denotes the ath-order FrFT.

$$y(t) = F^{-a}\left\{\boldsymbol{w}^H\left(F^a\{\boldsymbol{x}(t)\}\right)\right\} \tag{8}$$

where $F^a\{\cdot\}$ denotes the ath order FrFT. Since the structure of the beamformer is now changed, we have to recalculate the optimum weights. The goal is again to minimize the MSE between the desired signal and the output of the beamformer. We substitute the beamformer output (8) into the MSE (6) to be minimized and solve for the optimum orders. See [16] for details. The optimum weights that minimize the MSE are now given by

$$\boldsymbol{w}_{\rm opt} = R_{x_a}^{-1}\mathbf{r}_{x_a d} \tag{9}$$

where R_{x_a} is the covariance of the ath-order FrFTs of the signals arriving at the sensors, and $\boldsymbol{r}_{x_a d}$ is the cross-covariance between the ath-order FrFT of the desired signal and the FrFTs of the signals arriving at the sensors. The covariance R_{x_a} and the cross covariance $\mathbf{r}_{x_a d}$ should be known *a priori* in a moving source problem. On the other hand, in an active radar problem, we can calculate it since the signal transmitted is known to us, assuming a distribution for the parameters like range, radial velocity, and DOA of the target.

We can compute R_{x_a} and $\mathbf{r}_{x_a d}$ using the original covariances as follows:

$$\begin{aligned}
R_{x_a} =& R_{x_a}(t,t') \\
=& \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_a(t,t'')K_{-a}(t',t''')R_x(t'',t''')dt''\,dt''' \\
r_{x_a d} =& r_{x_a d}(t,t') \\
=& \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_a(t,t'')K_{-a}(t',t''')r_{xd}(t'',t''')\,dt''\,dt'''.
\end{aligned}$$

The previous discussion gives the optimum weights for beamforming in a certain fractional Fourier domain. We still need to answer the question as to which domain should be selected. The optimum FrFT order cannot be found analytically in general. Instead, we calculate the MSE we want to minimize [see (6)] for different as, and select the one that yields the smallest error. We can scan values of $a \in [-1, 1]$ using a spacing as close as we wish and make fine adjustment if necessary.

The proposed method reduces to the ordinary minimum MSE beamforming in the spatial domain for $a = 0$ and to the minimum MSE beamforming in the frequency domain for $a = 1$. In Section IV, we show that in many problems, the optimum order is different than 1 or 0; thus, smaller errors can be obtained when the generalized method we propose is used.

IV. Numerical Examples: Application to Moving Sources

We demonstrate that the proposed method yields improved results, that is smaller MSE, in a moving source scenario. We give three numerical examples for stationary, moving, and accelerating sources. The

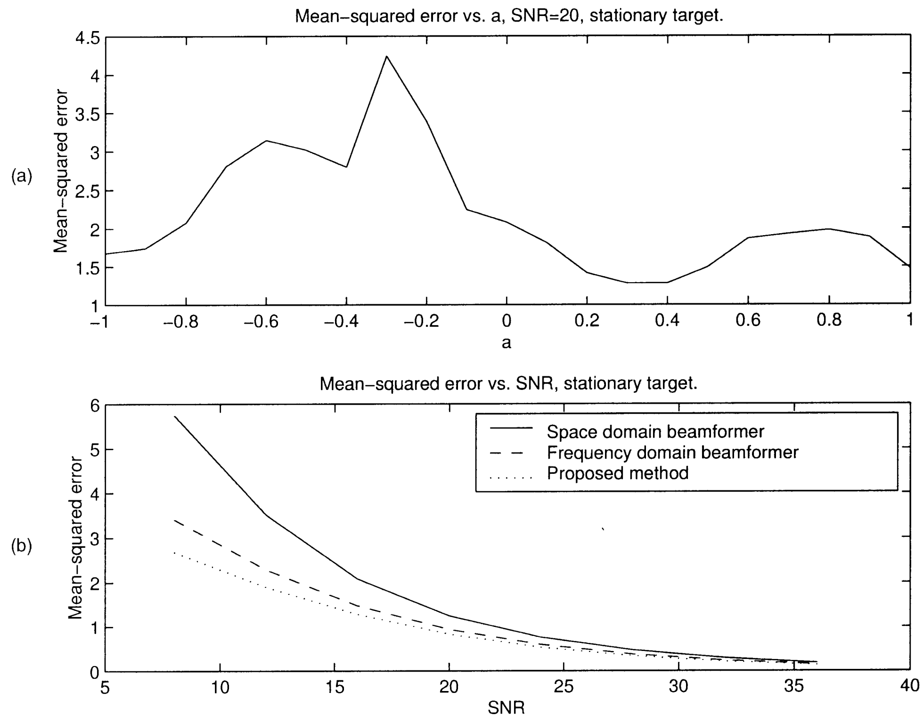

Fig. 4. Error plots for a stationary source. (a) MSE for the FrFT beamformer for different values of a. (b) Comparison of the MSE for the FrFT beamformer $(a = a_{\rm opt})$, space domain beamformer $(a = 0)$, and frequency domain beamformer $(a = 1)$.

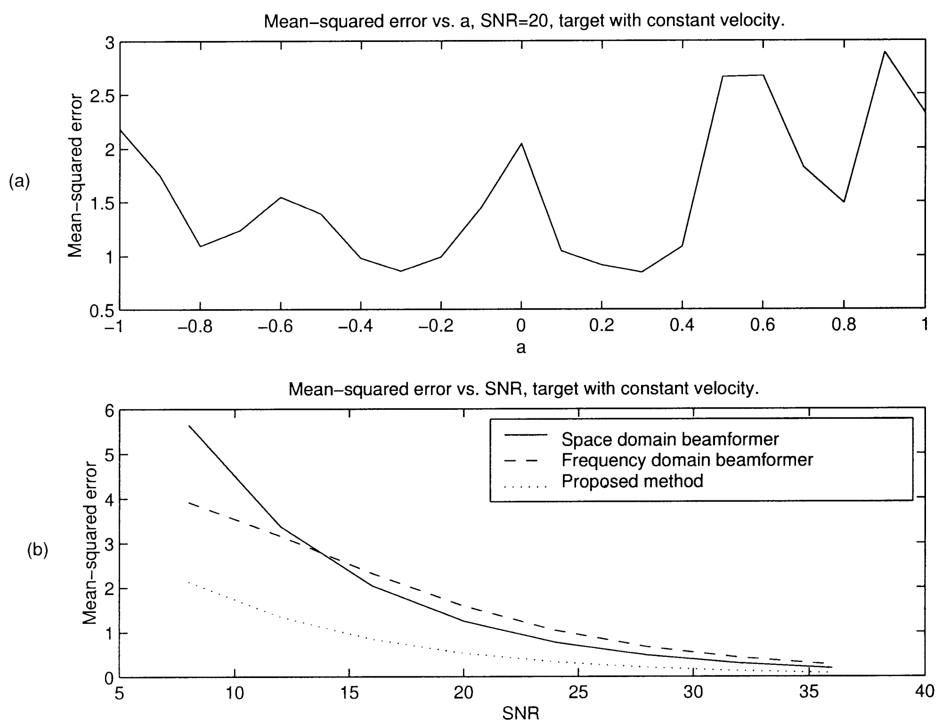

Fig. 5. Same as in Fig. 4 but for a moving source with constant velocity.

source is in the far field and emits an electromagnetic sinusoid with frequency $f = 100$ kHz; therefore, the wavelength λ is approximately 3 m. We assume additive Gaussian noise and use five linearly spaced passive sensors separated by half wavelength, and only instantaneous measurements are used, without delays. The source signal is assumed to be stochastic with known second-order statistics. In the second example, where the source is moving, we choose the velocity of the source to be 100 m/s in a direction perpendicular to the array line. In the third example, the source accelerates from 60 to 120 m/s during the measurement interval with an acceleration of 6 m/s^2 in the same direction. Figs. 4–6 show two figures of the MSE (6) for each scenario: (a) as a function of the FrFT order a and the (b) as a function of SNR for beamformers in the space domain $(a = 0)$, frequency domain $(a = 1)$, and the proposed method for the optimum order.

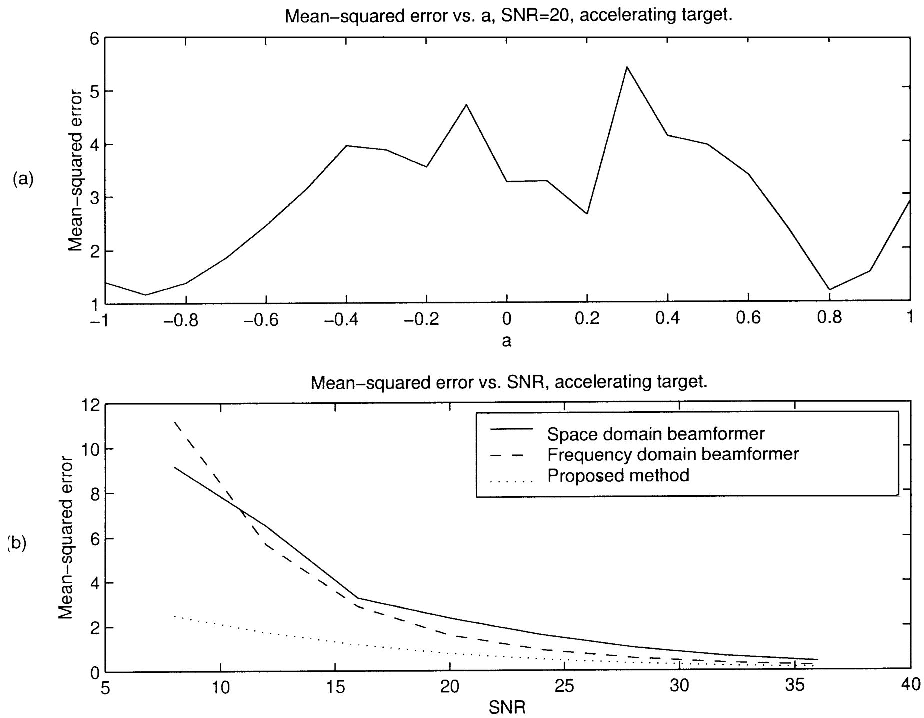

Fig. 6. Same as in Fig. 4 but for an accelerating source.

TABLE I
MSE FOR THE THREE SOURCE CASES AND THREE BEAMFORMER DOMAINS, SNR $= 20$

	MSE		
Domain	Stationary source	Const. velocity source	Accelerating source
$a = a_{\rm opt}$	1.26	0.84	1.07
$a = 0$	1.96	2.11	3.12
$a = 1$	1.48	2.37	2.85

Fig. 4(a) shows that the optimum order for the stationary sources is $a = 0.3$, Fig. 5(a) shows that for the moving source, it is $a = -0.3$, and Fig. 6(a) shows that for the accelerating source, it is $a = 0.8$; observe that all optimum orders are different than 0 and 1 (corresponding to standard space and frequency domain beamformers). The improvements in the performance can be easily observed in these plots, especially for low SNR. In addition, one can see that the improvement is generally larger in the moving and accelerating sources by comparing Figs. 4(b)–6(b). This is due to the fact that the FrFT is more effective in cases where chirp signals are involved, and the accelerating source produces a chirp signal due to the Doppler effect. The slight improvements in low SNR for the stationary source is due to the high level of noise, making it possible to extract the signal in some domain rather than $a = 0$ or $a = 1$, depending on the realization.

We summarize the results in Table I, for easy reference, where we present the MSE values for SNR $= 20$. The first row of the table shows the MSE for the proposed method for all three cases, i.e., stationary, constant velocity, and accelerating source. The second row shows the error when the spatial beamformer is used. The next row similarly shows the MSE when the frequency domain beamformer is used. Observe that the improvement in MSE for the accelerating source case is as much as 65.7% compared with the spatial beamformer and 62.5% compared with the frequency domain beamformer for SNR $= 20$.

V. CONCLUSION

We have proposed a method of beamforming using the FrFT. The method was shown to be especially useful (yielding smaller errors) for chirp signals, for instance, in moving source problems, where the Doppler effect produces chirp signals when the sinusoidal source is accelerating, and causes a frequency shift when the velocity of the source is constant. This method can be useful also in active radar when we chirp signals are transmitted to the target. Finding the optimum order using numerical or iterative methods rather than scanning could eliminate the need for trying many a values; hence, it is of interest for a future research. In addition, as another open research area, the proposed method can be further generalized to more complex filtering schemes such as multistage [12] or multichannel filtering [14], [15] schemes rather than a single fractional Fourier domain filtering. Another interesting subject to be investigated is the relationship between the array geometry and filtering schemes to be used. Additionally, beamforming in the time-frequency plane, which is going beyond FrFT, can be useful in certain applications.

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Blind MIMO FIR Channel Identification Based on Second-Order Spectra Correlations

Ivan Bradaric, Athina P. Petropulu, and Konstantinos I. Diamantaras

***Abstract*—We consider a problem of identifying a multiple-input multiple-output (MIMO) finite impulse response (FIR) system excited by colored inputs with known statistics. We propose a new, nonlinear optimization-based method that involves the power spectra and cross-spectra of the system output. The proposed algorithm is tested for the case of cyclostationary inputs (CDMA scenario) and stationary inputs (SDMA scenario). Simulation results indicate that the proposed scheme works well, even for large order systems, and is robust to noise and channel length mismatch.**

***Index Terms*—Blind channel estimation, frequency domain estimation, MIMO system.**

I. Introduction

We consider the problem of identifying a P-input M-output linear time-invariant finite impulse response (FIR) system based on the system output. This problem is referred to as blind system identification and appears in many contexts, such as speech restoration in the presence of competing speakers, bioengineering, and multiuser multiaccess communications.

Identification in the case of spatially independent and temporally white inputs has been approached mainly using higher order statistics of the system output [3], [4], [10], [14]. Identification for inputs that are stationary nonwhite but can be modeled as linear processes has been approached using either second [6], [8], [15] or higher order statistics [12], [20]. In those cases, if the input statistics are unknown, the inputs can be decoupled, but there is a remaining shaping filter ambiguity in each input. Under certain conditions, the shaping filter ambiguity can be removed [2], [6], [12].

The identification problem can be simplified if some information about the input is available or if the inputs can be manipulated. Assuming that the input signals have the same known period, and under certain channel conditions, a method for separation of finite alphabet signals has been proposed in [16]. In [5], by inducing cyclostationarity in the input signals, a closed-form solution was obtained based on exclusively second-order statistics. Second-order cyclostationary statistics has also been used in [1] for blind multiple-input–multiple-output (MIMO) identification in OFDM-based multiantenna systems. In [7], a special structure was imposed on each input and the resulting MIMO problem with colored inputs was solved using second-order statistics.

In this paper, we consider the problem of identifying the impulse response of an FIR MIMO system based on the system output. The system inputs are stationary or cyclostationary and nonwhite with known correlations. There are several practical cases of MIMO problems where the statistics of the inputs are known, such as code-division multiple asccess (CDMA) systems [11], [17], [19] or spatial division

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