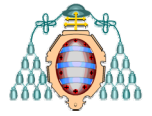


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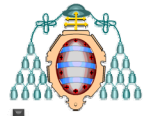
# Sintonización de la acción diferencial

Sistemas Automáticos– Tema 10



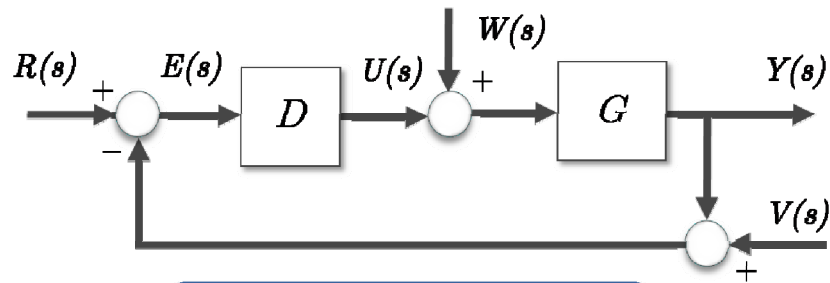
# Contenidos del tema

- ▶ La acción proporcional diferencial
- ▶ Efecto de la ganancia diferencial
- ▶ PD real o red de adelanto de fase
- ▶ Sintonización del PD real por medio del lugar de las raíces
- ▶ Sintonización de la red de adelanto de fase por medio de la función de lazo



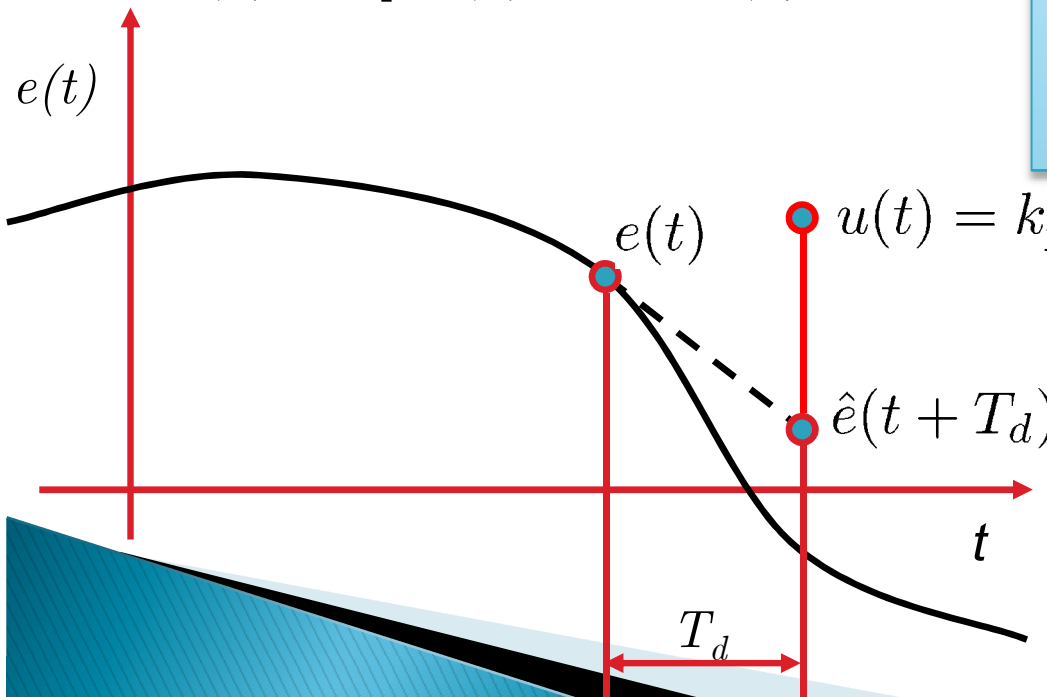
# La acción proporcional-diferencial

## Propiedades



$$D(s) = k_p + k_d s$$

$$U(s) = k_p E(s) + k_d s E(s)$$



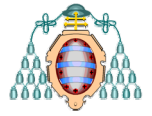
- Se basa en la predicción lineal del valor del error dentro de  $T_d$  segundos  $\rightarrow$  es *anticipativa*.
- Esta predicción se basa en la pendiente del error (derivada).
- Su carácter anticipativo hace que, en general, mejore la dinámica de la respuesta.

$$u(t) = k_p \cdot \hat{e}(t + T_d) = k_p \cdot e(t) + k_p \cdot T_d \cdot \frac{de(t)}{dt}$$

parte  
proporcional

parte  
diferencial

$$k_d = k_p \cdot T_d$$



# Efecto de la ganancia diferencial

$$D(s) = k_p + k_d s = k_p \left( 1 + \frac{k_d}{k_p} s \right) = k_p (1 + T_d s)$$

Polos y ceros del regulador

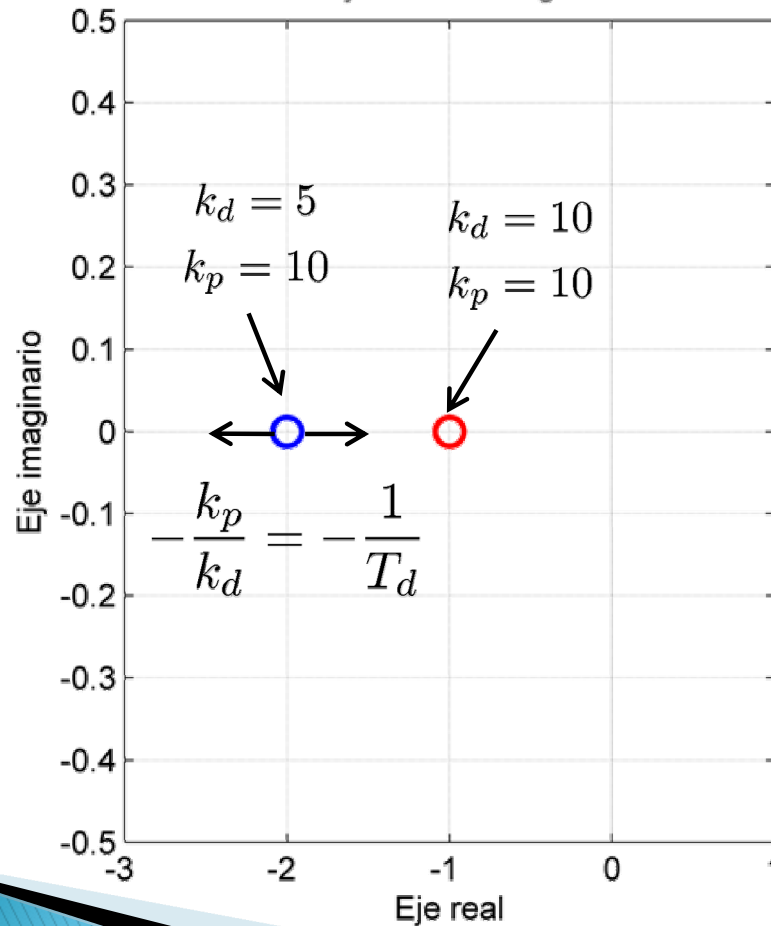
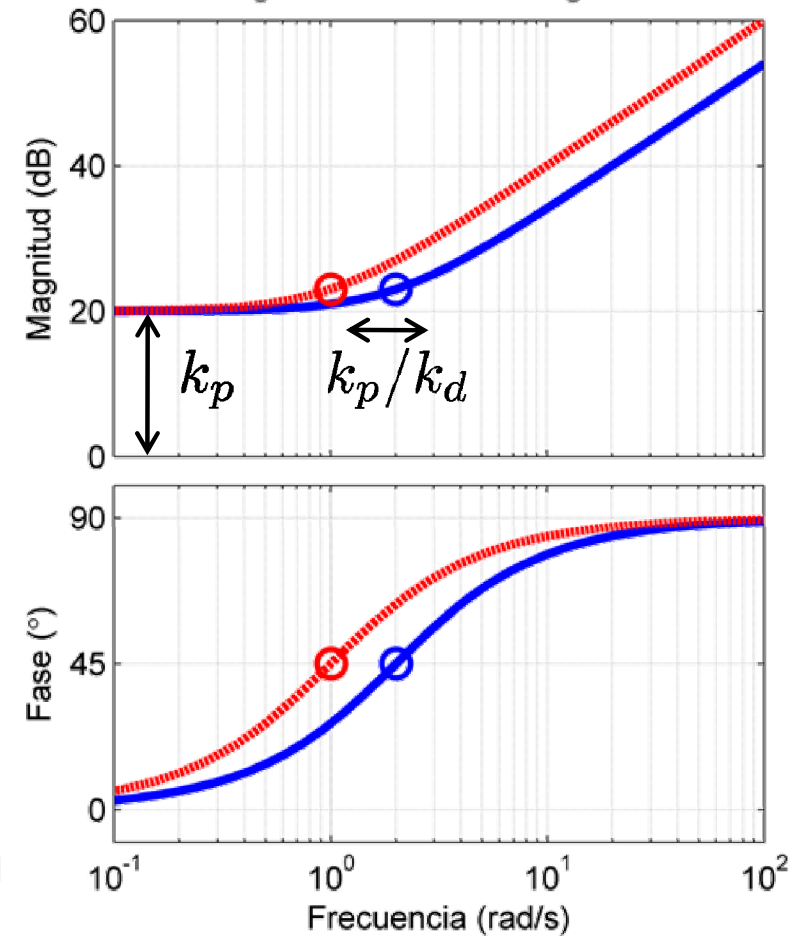
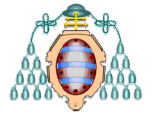


Diagrama de Bode del regulador





# PD real o red de adelanto de fase

$$D(s) = k \frac{s + z}{s + p}, \quad p > z$$

$$D(s) = k_p \frac{Ts + 1}{\alpha Ts + 1}, \quad 0,1 < \alpha < 1$$

Polos y ceros del regulador

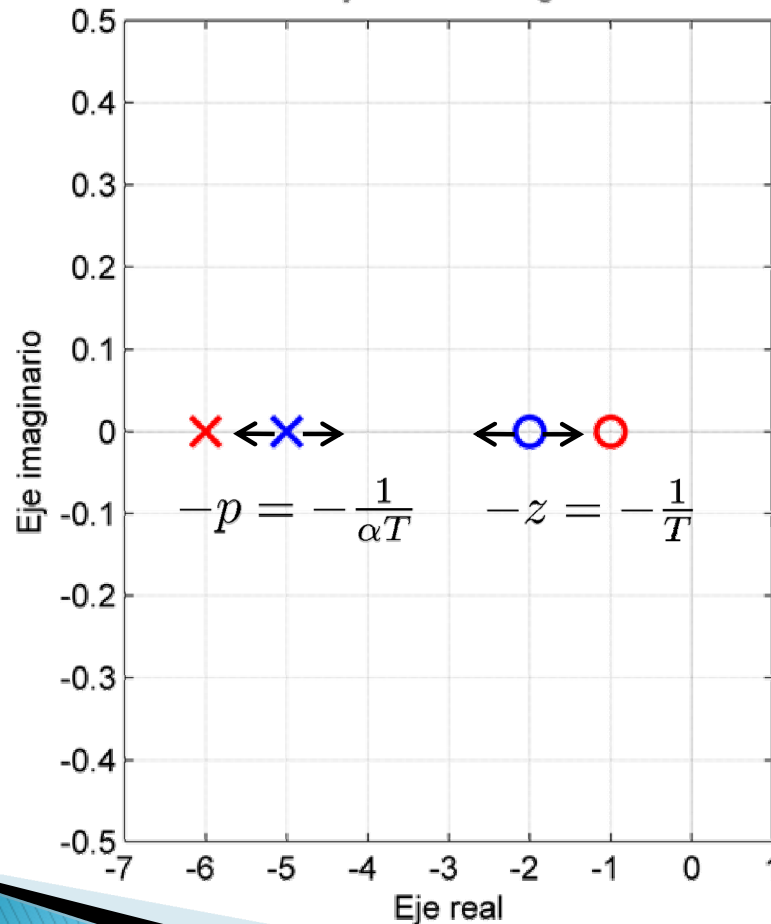
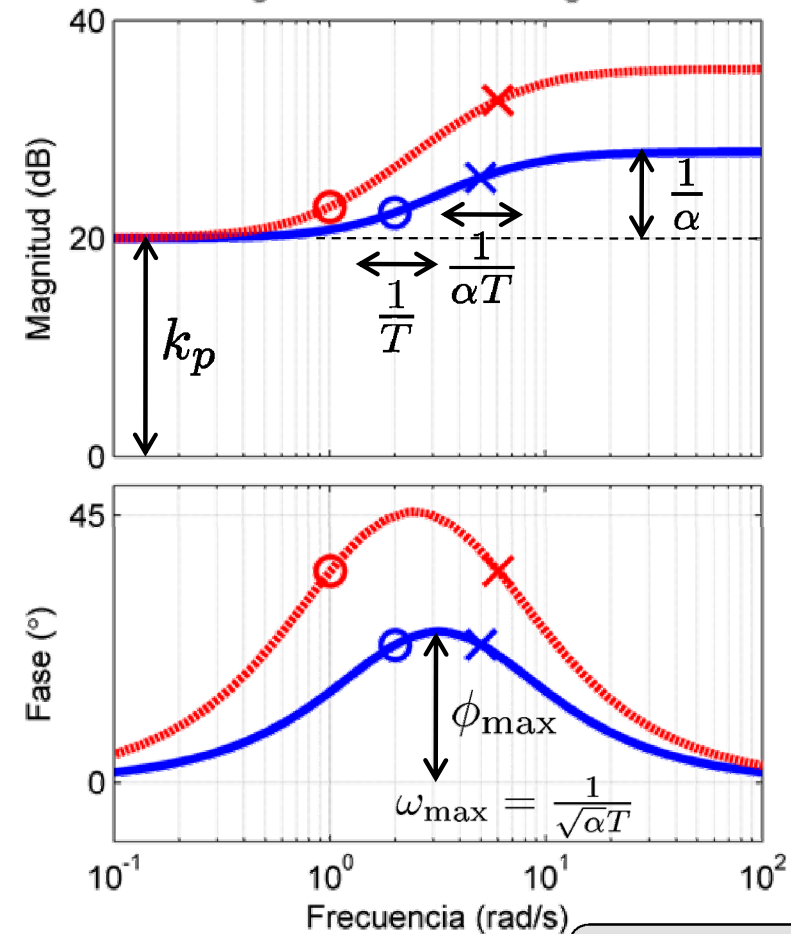
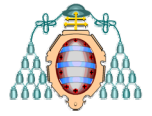


Diagrama de Bode del regulador



$$\alpha = \frac{1 - \sin \phi_{\max}}{1 + \sin \phi_{\max}}$$



# Sintonización: lugar de las raíces

$$D(s) = k_p$$

$$G(s) = \frac{6(s+8)}{(s+1)(s+3)(s+6)}$$

$$H(s) = 1$$

Especificaciones:

$$M_p \leq 30\%$$

$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$M_p = e^{-\pi \tan \theta}$$

$$e^{-\pi \tan \theta} \leq 0,3$$

$$\theta \geq 21^\circ$$

$$e_p = \lim_{s \rightarrow 0} \frac{1 + DGH(s) - DG(s)}{1 + DGH(s)}$$

$$e_p = \frac{1}{1 + D(0)\frac{8}{3}}$$

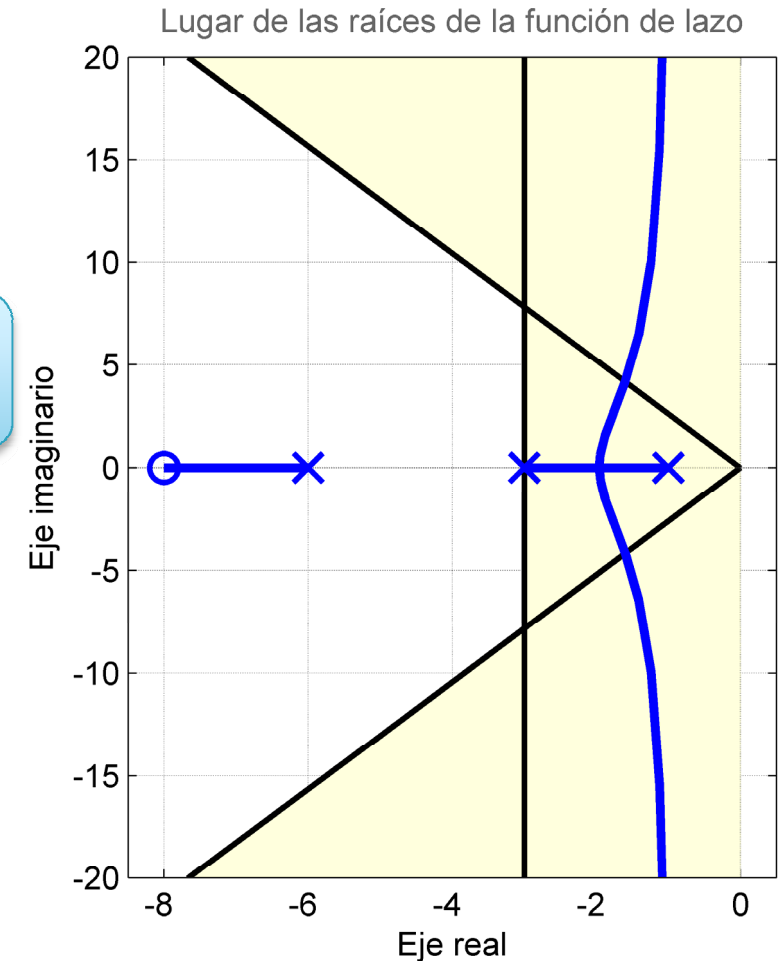
$$\frac{1}{1 + D(0)\frac{8}{3}} \leq 0,15$$

$$D(0) \geq 2,125$$

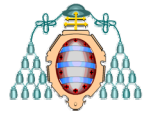
$$t_s = \frac{3}{\sigma}$$

$$\frac{3}{\sigma} \leq 1$$

$$\sigma \geq 3$$



Se necesita modificar el lugar de las raíces:  
regulador PD



# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

Especificaciones:

$$M_p \leq 30\%$$

$$t_s(5\%) \leq 1s$$

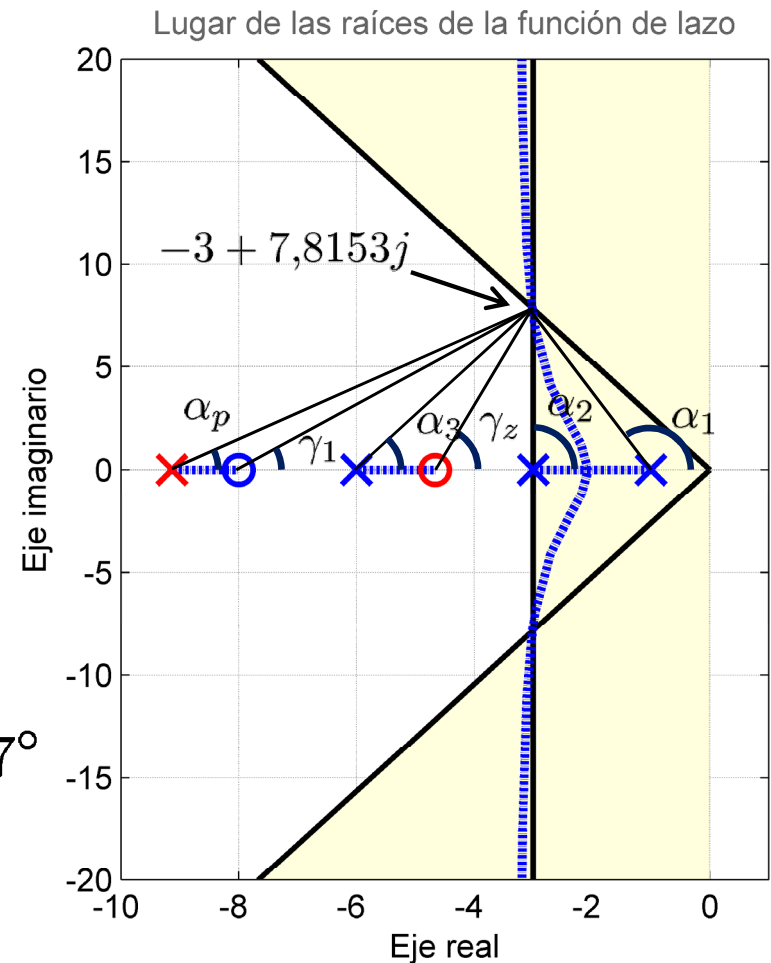
$$e_{rp} \leq 15\%$$

$$\sum_{j=1}^m \gamma_{zj} - \sum_{i=1}^n \alpha_{pi} = 180^\circ$$

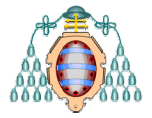
$$(\gamma_z - \alpha_p) = 180^\circ - (\gamma_1 - \alpha_1 - \alpha_2 - \alpha_3)$$

$$\alpha_1 = 104,36^\circ \quad \alpha_2 = 90^\circ \quad \alpha_3 = 69^\circ \quad \gamma_1 = 57,37^\circ$$

$$(\gamma_z - \alpha_p) = 26^\circ$$



Existen distintas posibilidades de colocar el polo y el cero



# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

Especificaciones:

$$M_p \leq 30\%$$

$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

$$(\gamma_z - \alpha_p) = 26^\circ$$

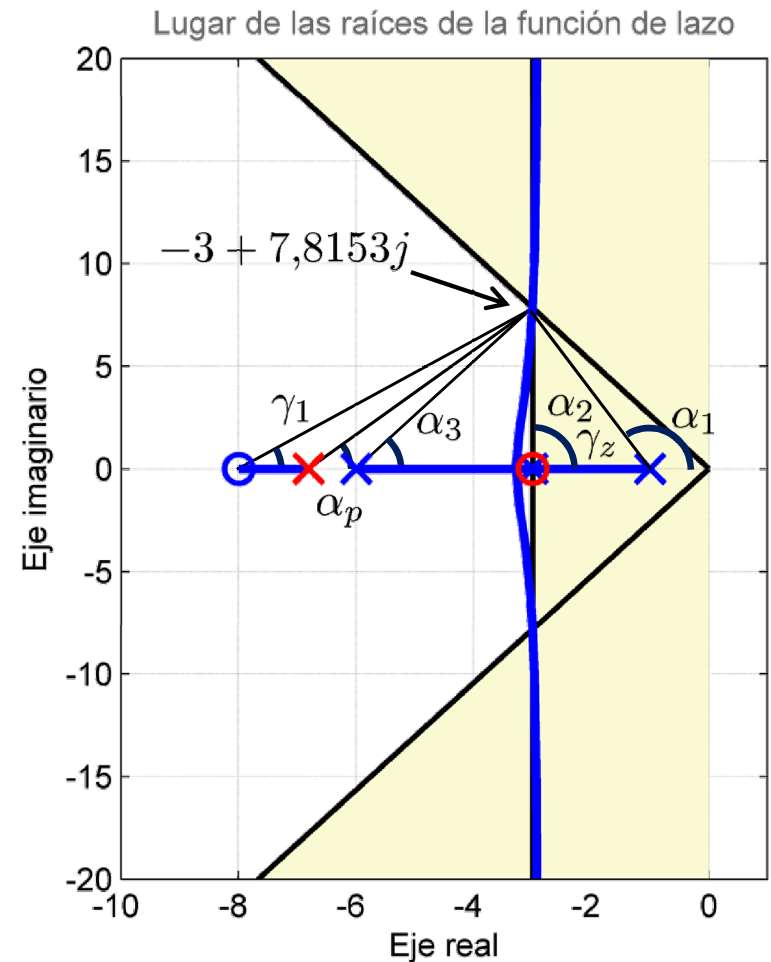
$$\gamma_z = 90^\circ \longrightarrow \alpha_p = 90^\circ - 26^\circ = 64^\circ$$

$$p = 6,81, z = 3$$

$$K = \frac{d_{p1} \cdot d_{p2} \cdot d_{p3} \cdot d_p}{d_{z1} \cdot d_z} = 63,20$$

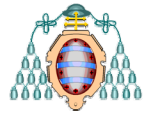
$$k = 10,53$$

Criterio de la vertical



$$D(0) = 10,53 \frac{3}{6,81} = 4,64 > 2,125$$





# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

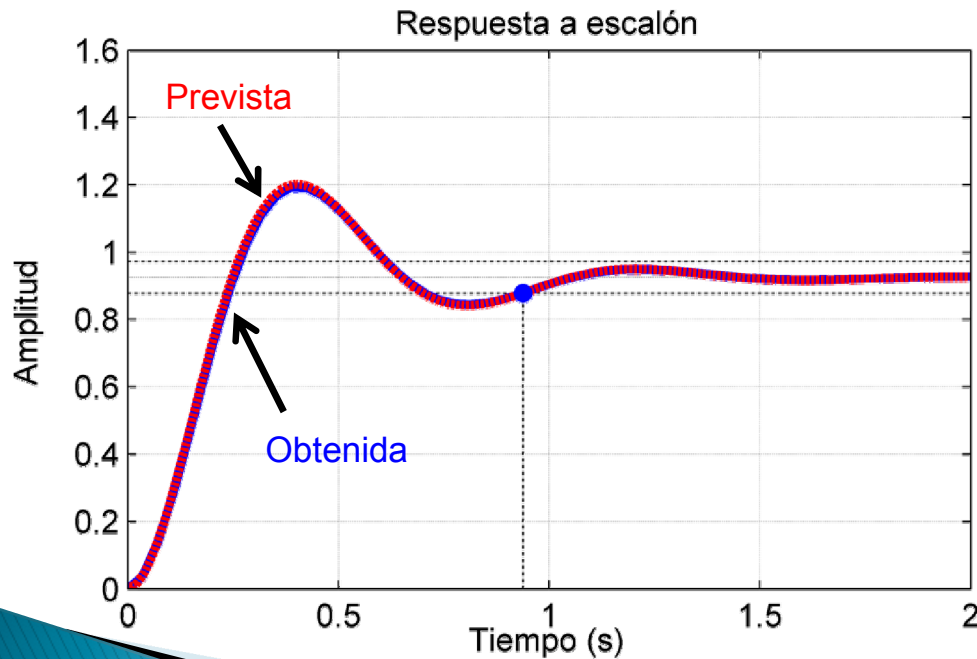
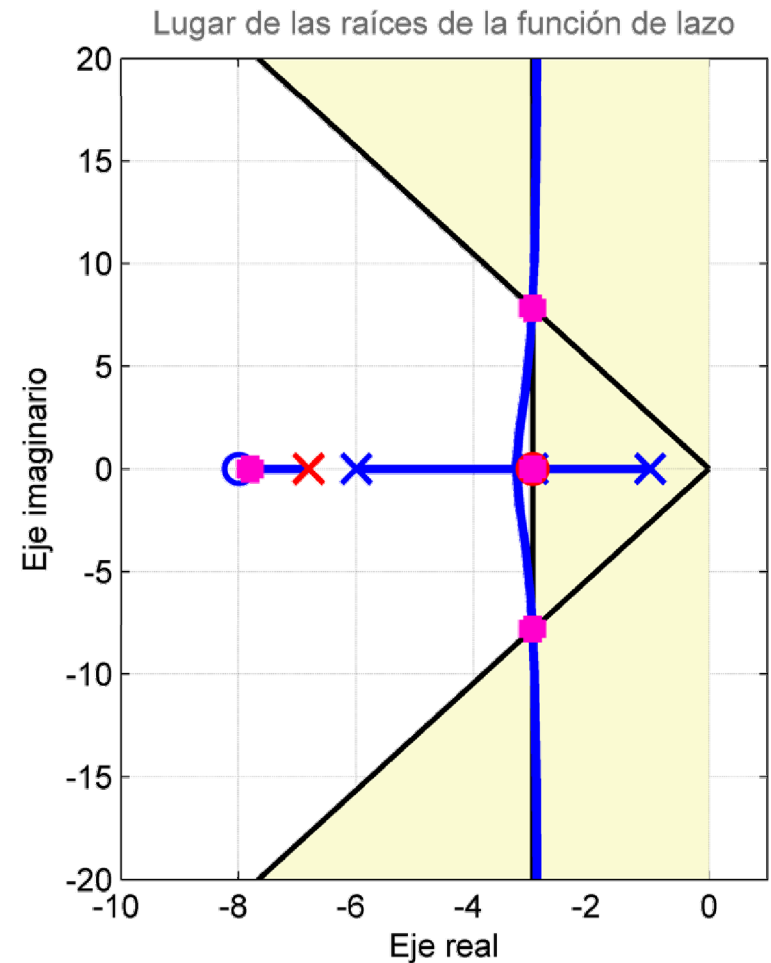
Especificaciones:

$$M_p \leq 30\%$$

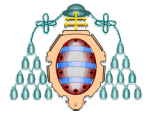
$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

Criterio de la vertical



$$D(s) = 10,53 \frac{s + 3}{s + 6,81}$$



# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

Especificaciones:

$$M_p \leq 30\%$$

$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

Cancelación polo menos significativo

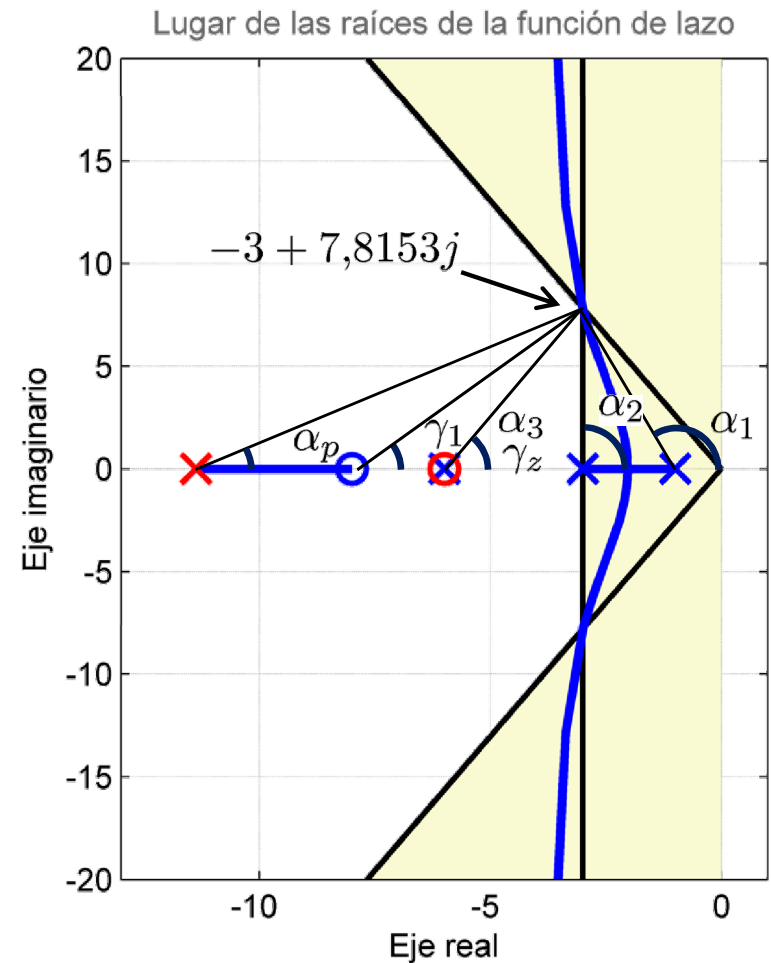
$$(\gamma_z - \alpha_p) = 26^\circ$$

$$\gamma_z = \alpha_3 \longrightarrow \alpha_p = 69^\circ - 26^\circ = 43^\circ$$

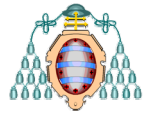
$$p = 11,38, z = 6$$

$$K = \frac{d_{p1} \cdot d_{p2} \cdot d_{p3} \cdot d_p}{d_{z1} \cdot d_z} = 77,8$$

$$k = 12,97$$



$$D(0) = 12,97 \frac{6}{11,38} = 6,83 > 2,125$$



# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

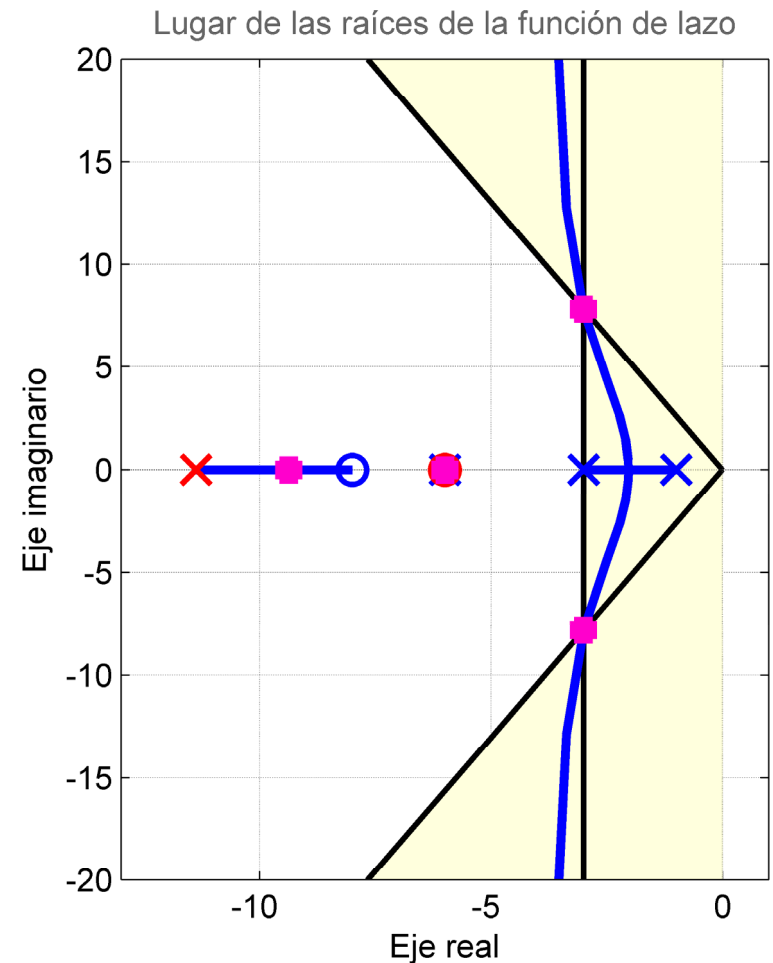
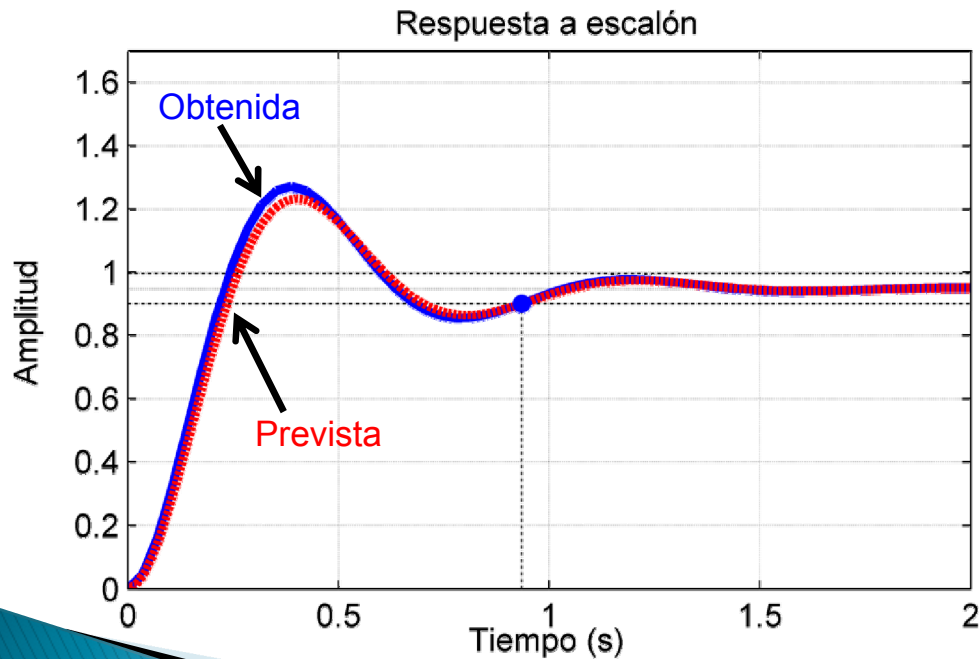
Especificaciones:

$$M_p \leq 30\%$$

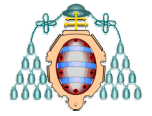
$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

Cancelación polo menos significativo



$$D(s) = 12,97 \frac{s + 6}{s + 11,38}$$



# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

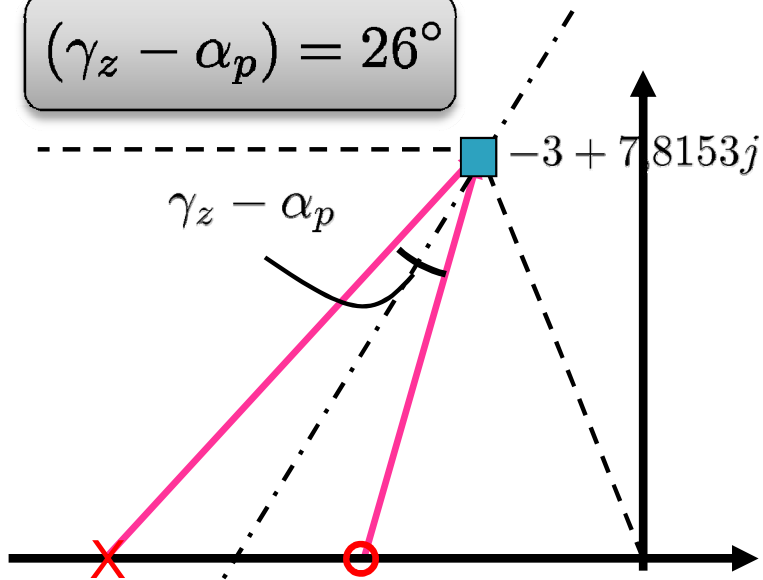
Especificaciones:

$$M_p \leq 30\%$$

$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

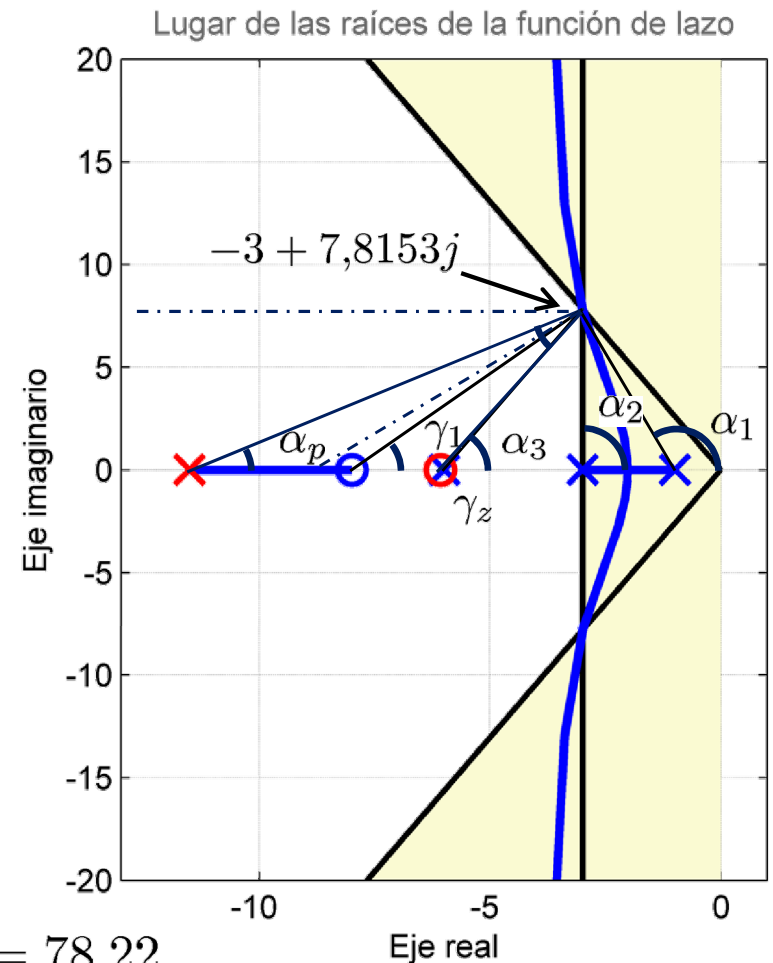
$$(\gamma_z - \alpha_p) = 26^\circ$$



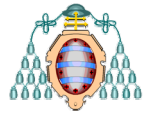
$$p = 11,52, z = 6,075 \quad K = \frac{d_{p1} \cdot d_{p2} \cdot d_{p3} \cdot d_p}{d_{z1} \cdot d_z} = 78,22$$

$$k = 13,03$$

Criterio de la bisectriz



$$D(0) = 13,03 \frac{6,075}{11,52} = 6,87 > 2,125$$



# Sintonización: lugar de las raíces

$$D(s) = k \frac{s + z}{s + p}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

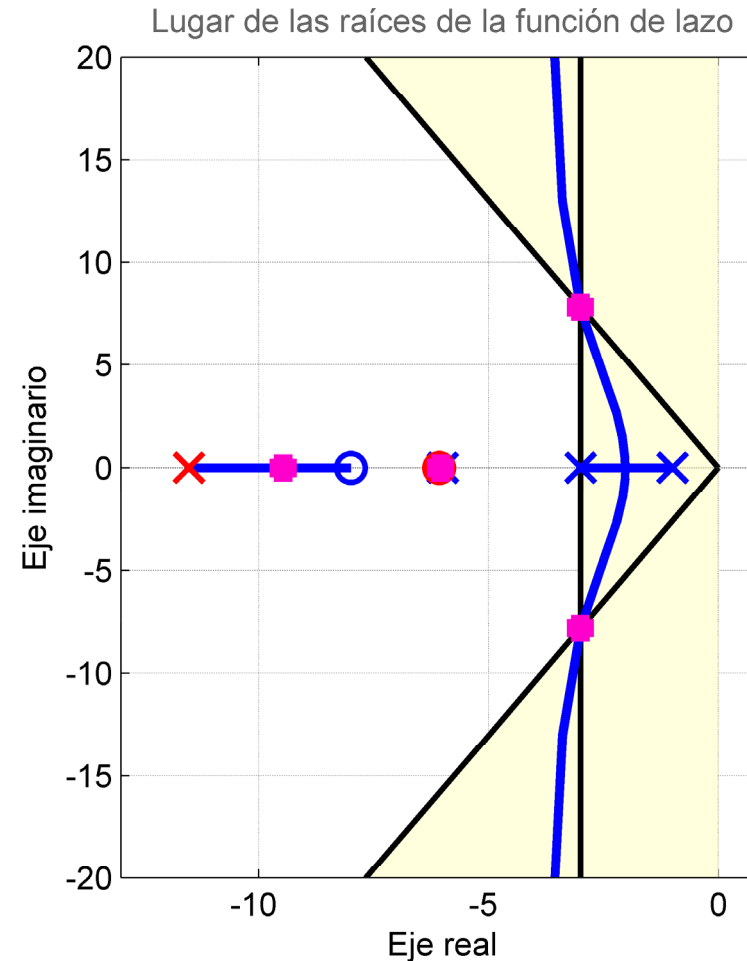
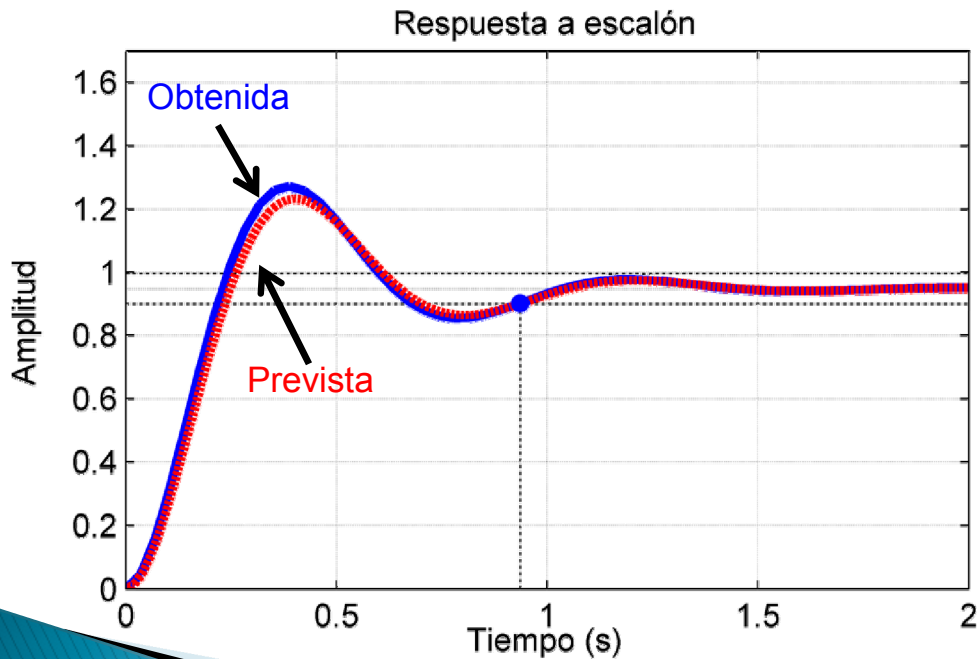
Especificaciones:

$$M_p \leq 30\%$$

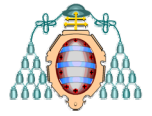
$$t_s(5\%) \leq 1s$$

$$e_{rp} \leq 15\%$$

Criterio de la bisectriz



$$D(s) = 13,03 \frac{s + 6,075}{s + 11,52}$$



# Sintonización: frecuencia

$$D(s) = k_p$$

$$G(s) = \frac{6(s+8)}{(s+1)(s+3)(s+6)}$$

$$H(s) = 1$$

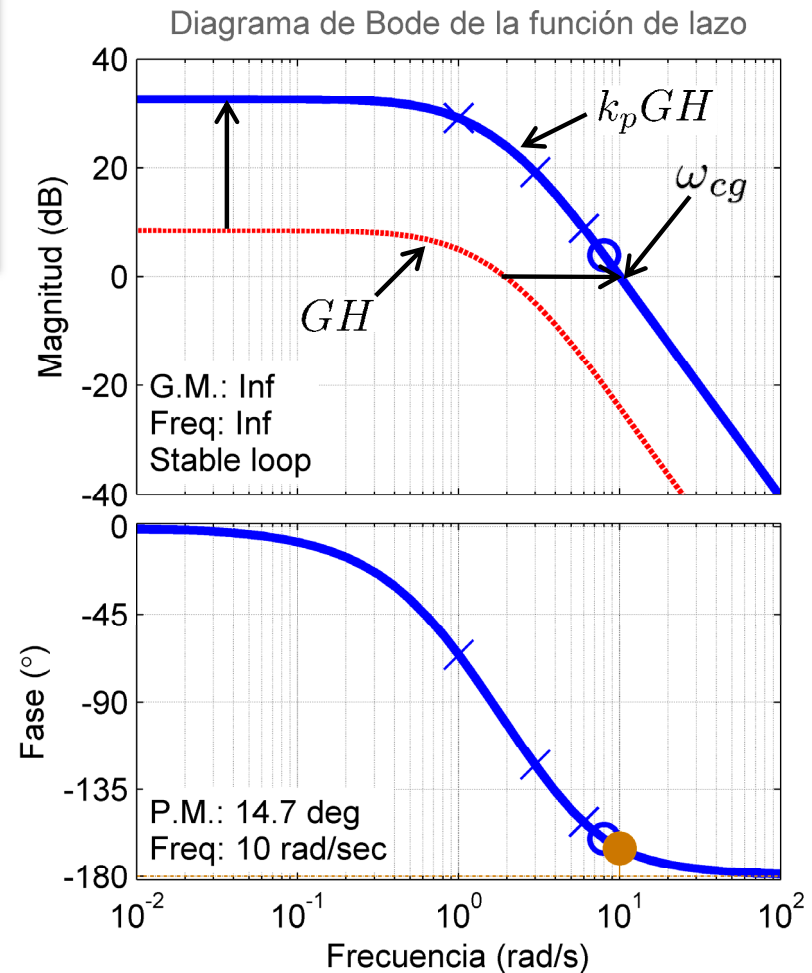
Especificaciones:

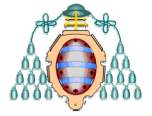
$$MF \geq 30^\circ$$

$$\omega_{bw} \approx 10 \text{ rad/s}$$

$$\omega_{bw} \approx \omega_{cg}$$

No se puede obtener el ancho de banda deseado manteniendo un adecuado margen de fase :  
regulador PD o red de adelanto de fase





# Sintonización: frecuencia

$$D(s) = k_p \frac{Ts + 1}{\alpha Ts + 1}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

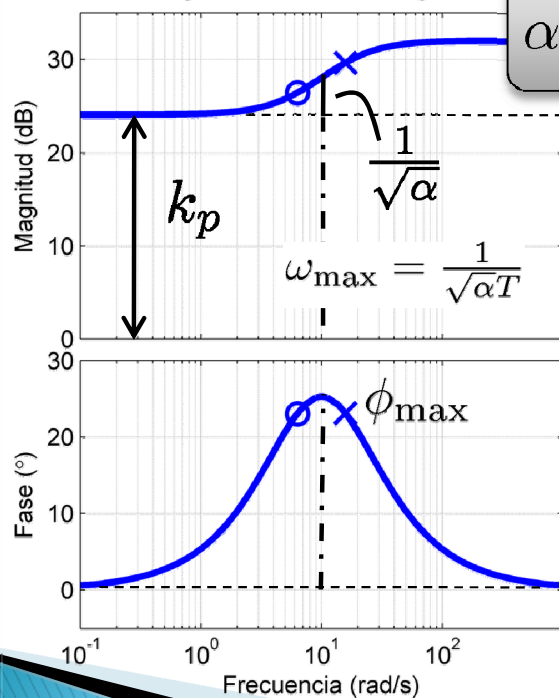
$$H(s) = 1$$

Especificaciones:

$$MF \geq 30^\circ$$

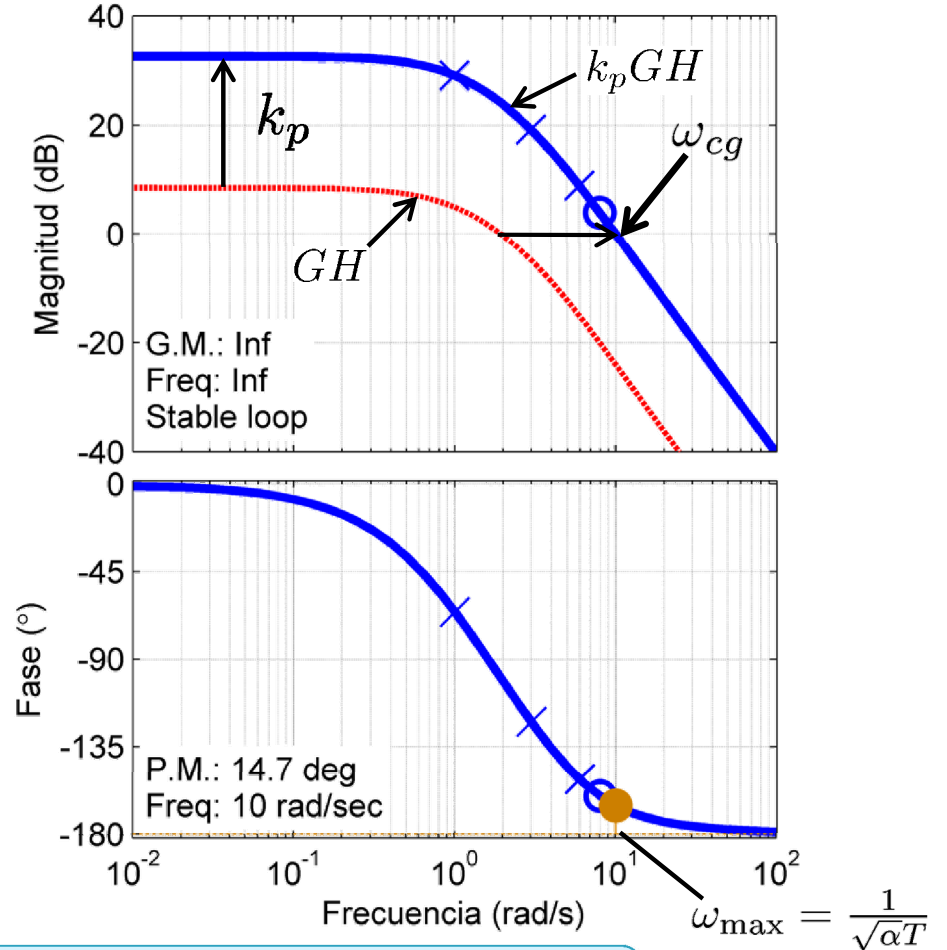
$$\omega_{bw} \approx 10 \text{ rad/s}$$

Diagrama de Bode del regulador



$$\alpha = \frac{1 - \sin \phi_{\max}}{1 + \sin \phi_{\max}}$$

Diagrama de Bode de la función de lazo

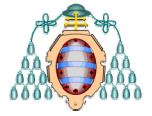


$$\phi_{\max} = MF_{\text{deseado}} - MF_{\text{actual}} + \phi_{\text{seguridad}}$$

$$\phi_{\max} = 30^\circ - 14,7^\circ + 10^\circ = 25,3^\circ$$

$$\alpha = \frac{1 - \sin 25,3^\circ}{1 + \sin 25,3^\circ} = 0,40$$

$$T = \frac{1}{\sqrt{\alpha} \cdot \omega_{\max}} = 0,1579$$



# Sintonización: frecuencia

$$D(s) = k_p \frac{Ts + 1}{\alpha Ts + 1}$$

$$G(s) = \frac{6(s + 8)}{(s + 1)(s + 3)(s + 6)}$$

$$H(s) = 1$$

Especificaciones:

$$MF \geq 30^\circ$$

$$\omega_{bw} \approx 10 \text{ rad/s}$$

$$D(s) = 16 \frac{0,1579s + 1}{0,0633s + 1}$$

Diagrama de Bode del sistema en lazo cerrado

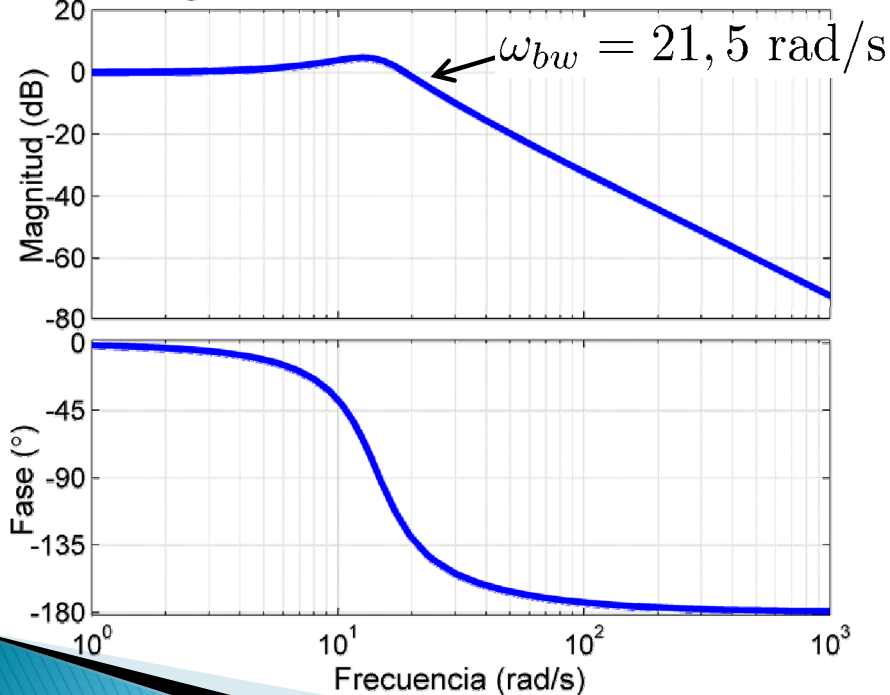


Diagrama de Bode de la función de lazo

