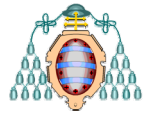


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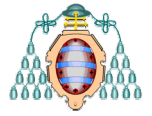
# Análisis del lazo de realimentación: referencias

Sistemas Automáticos– Tema 5

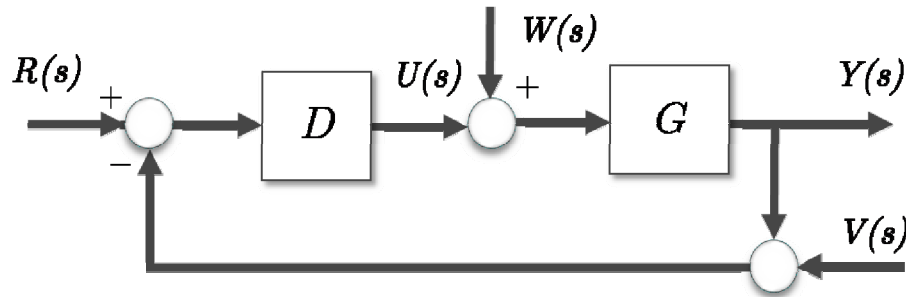


# Contenidos del tema

- ▶ Seguimiento de referencias
- ▶ Errores en régimen permanente
- ▶ Comportamiento dinámico



# Seguimiento de referencias



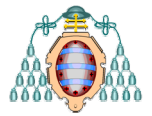
Sensibilidad  
complementaria

$$T = \frac{Y}{R} = \frac{DG}{1 + DG}$$

Sensibilidad

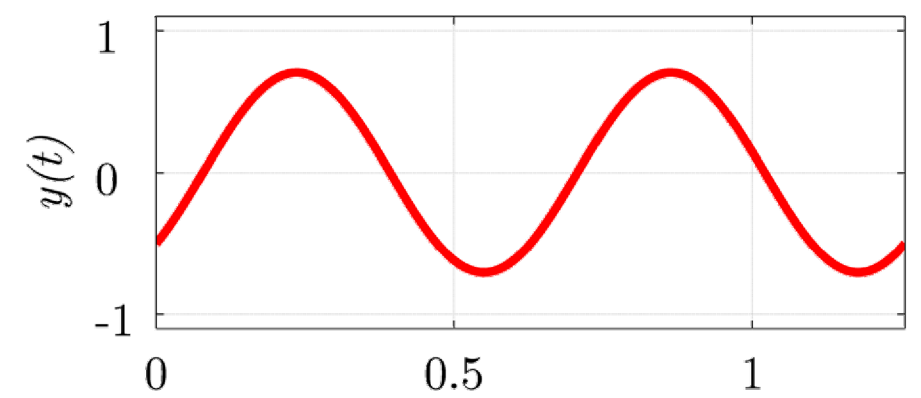
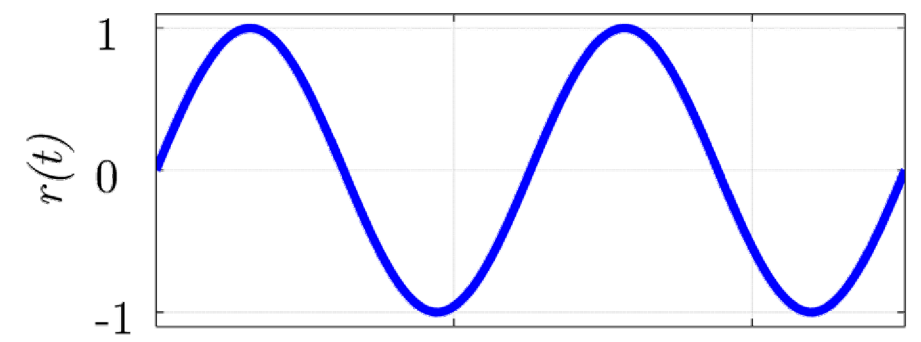
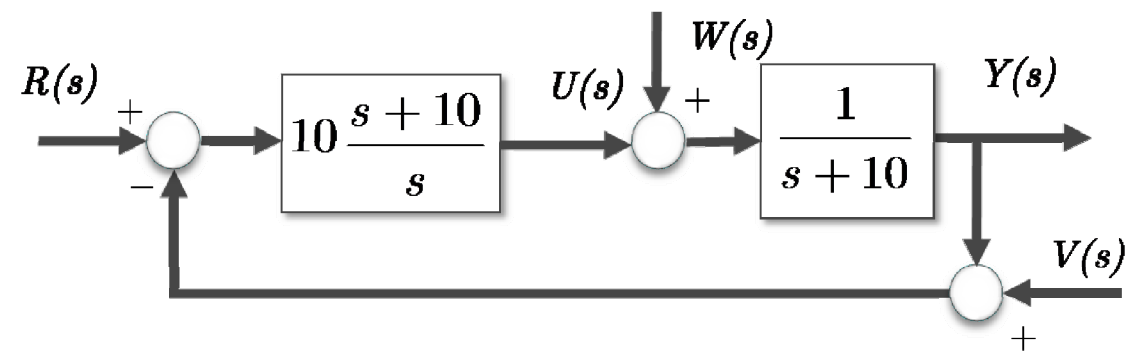
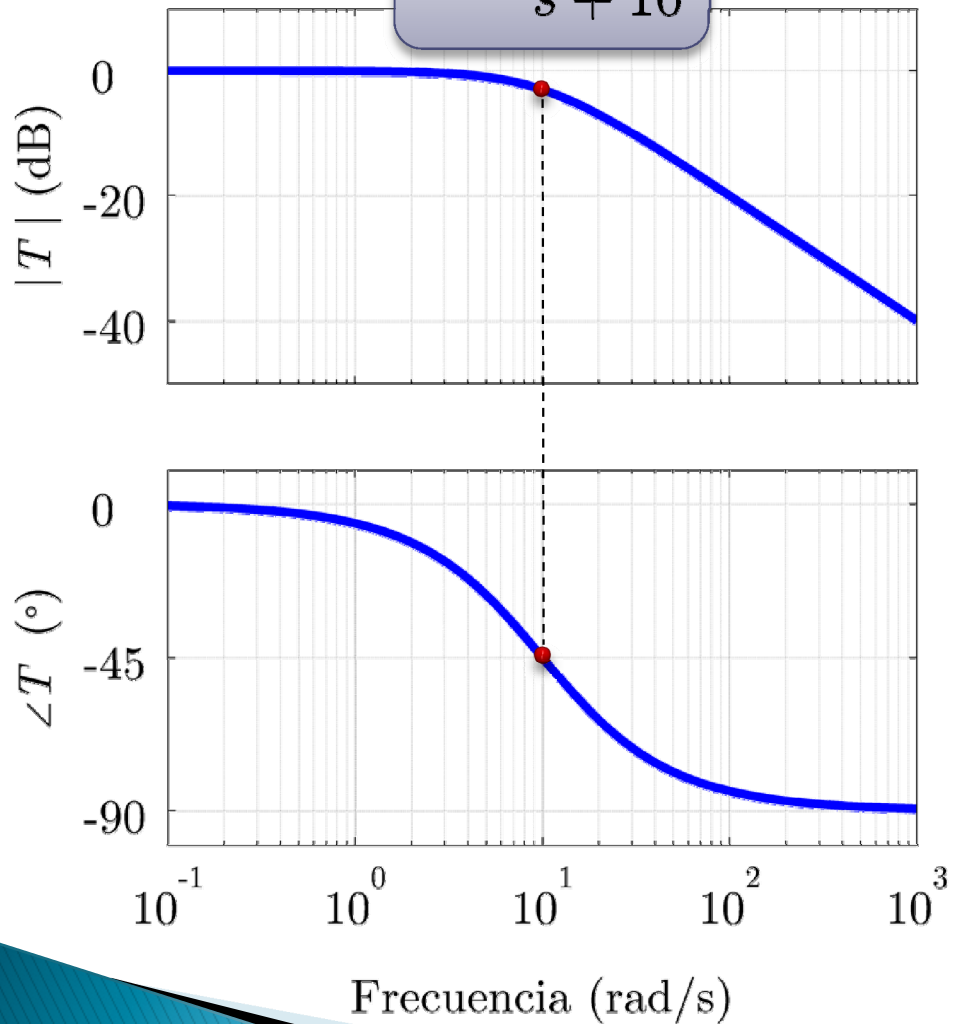
$$S = \frac{E}{R} = \frac{1}{1 + DG}$$

- ▶ Para analizar el seguimiento de referencias podemos utilizar tanto  $T$  como  $S$ .
- ▶  $T$  nos informa del valor de la salida  $Y$  dada una referencia  $R$ .
- ▶  $S$  nos da la diferencia entre la referencia  $R$  y la salida  $Y$ , es decir el error,  $E = R - Y$ .
- ▶ Se cumple que  $T + S = 1$

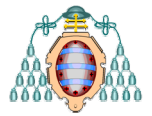


# Seguimiento de referencias

$$T = \frac{10}{s + 10}$$

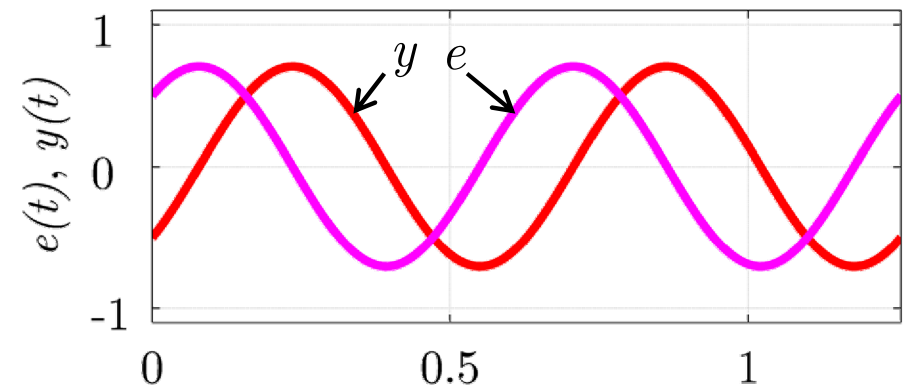
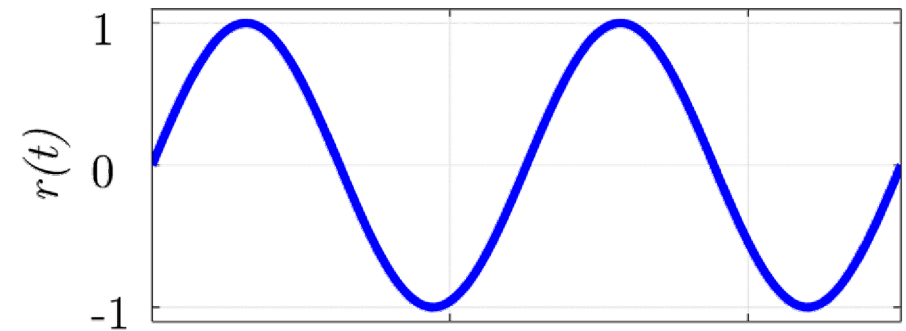
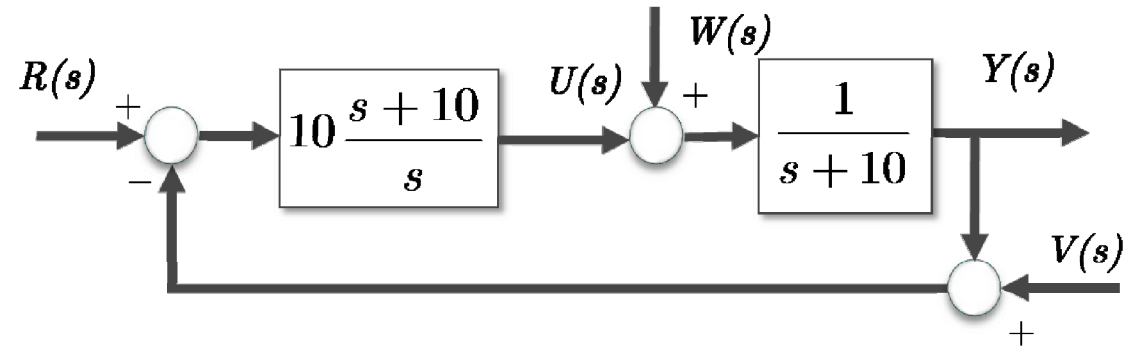
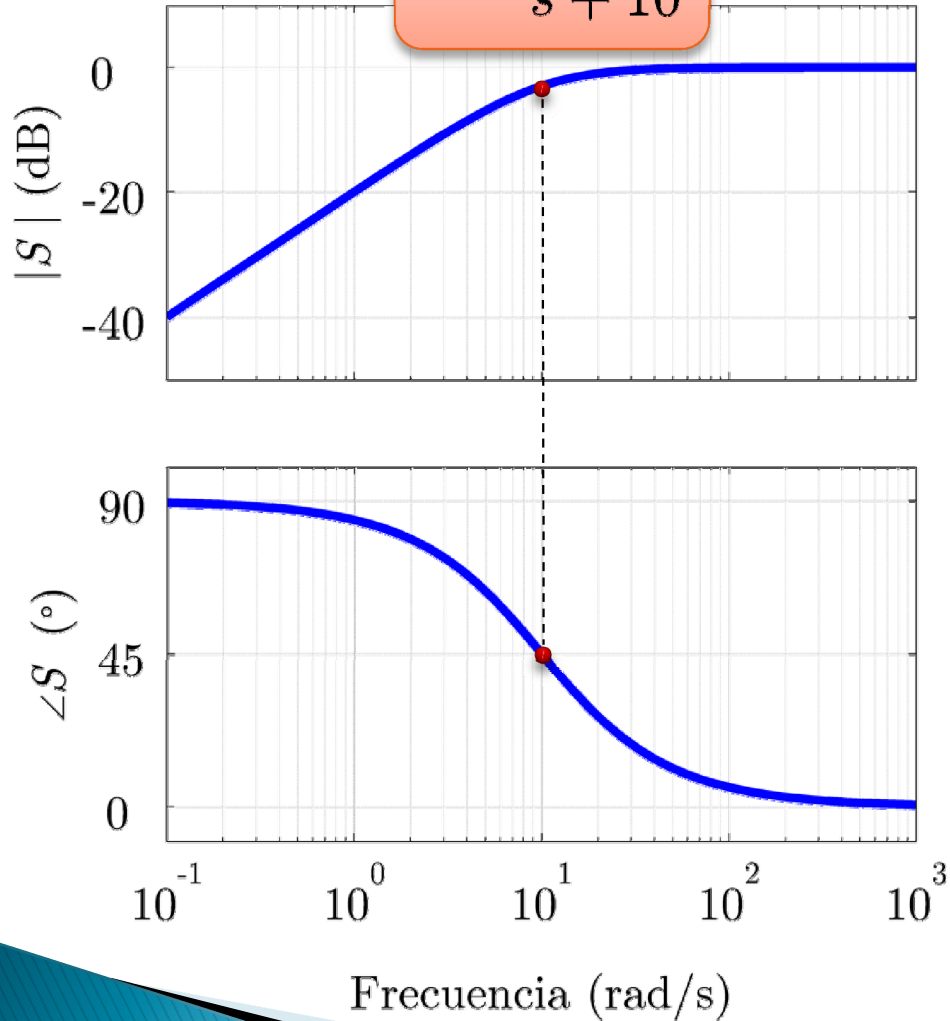


Tiempo (s)



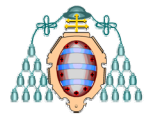
# Seguimiento de referencias

$$S = \frac{s}{s + 10}$$

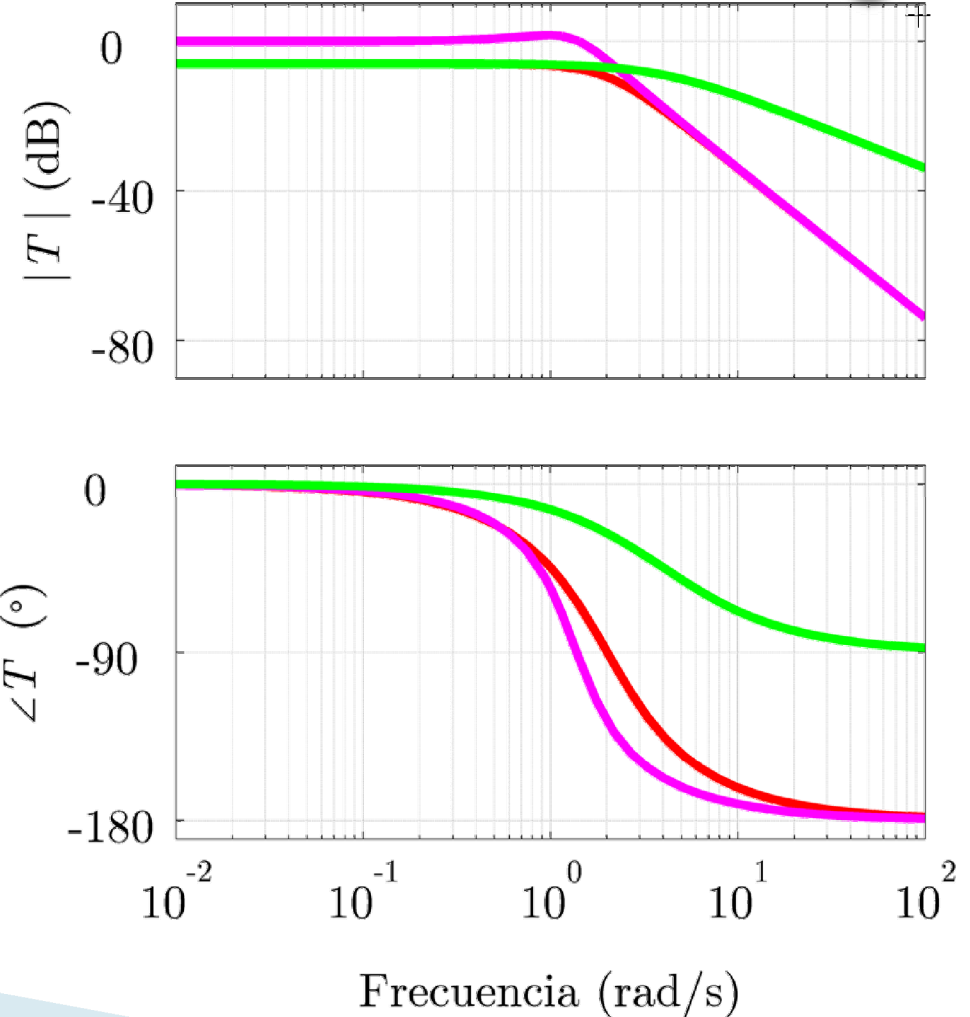
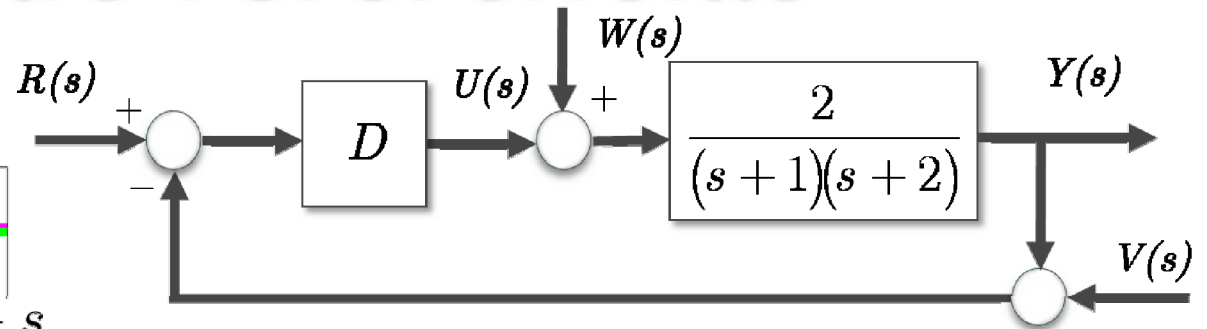
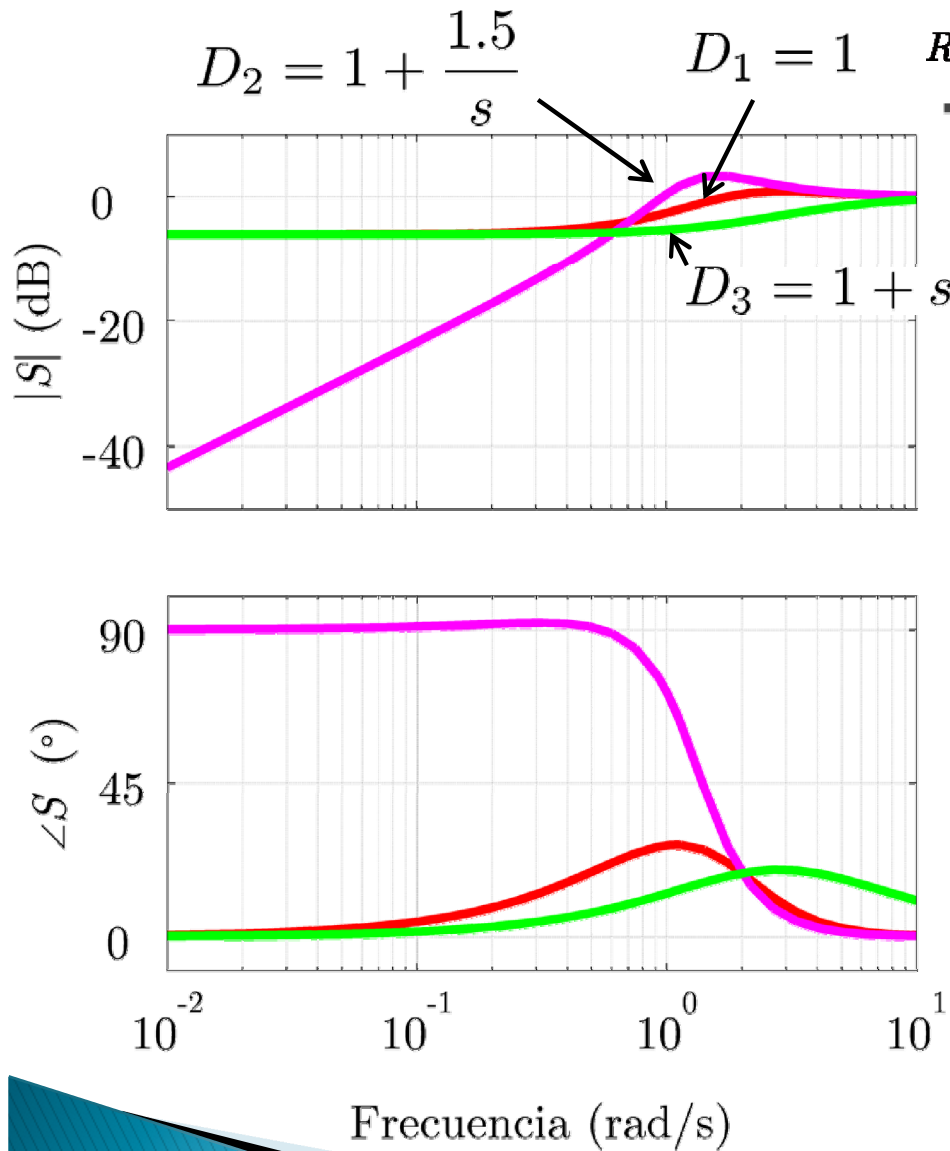


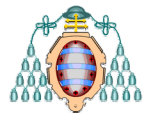
Tiempo (s)





# Seguimiento de referencias





# Errores en régimen permanente

- ▶ Se dice que un sistema es de tipo  $n$  (para seguimiento de referencias) cuando tiene error constante y no nulo en régimen permanente ante referencias de tipo polinomial de grado  $n$ .
- ▶ Las referencias de tipo polinomial de grado  $n$  son de la forma:

$$r(t) = k \cdot t^n$$

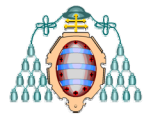
$$R(s) = \frac{k \cdot n!}{s^{n+1}}$$

- ▶ Ejemplos de señales de referencia de tipo polinomial:

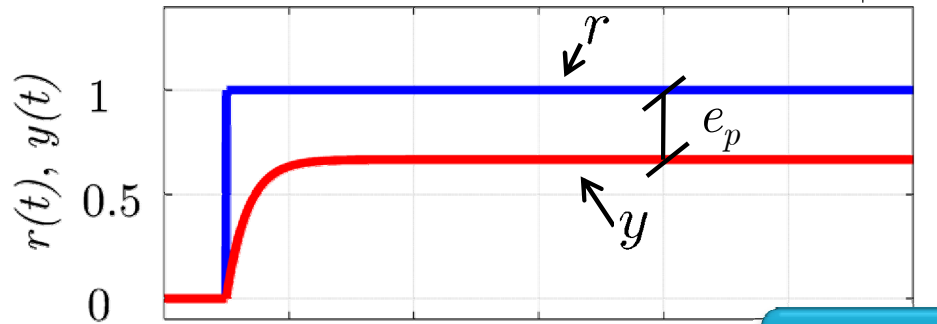
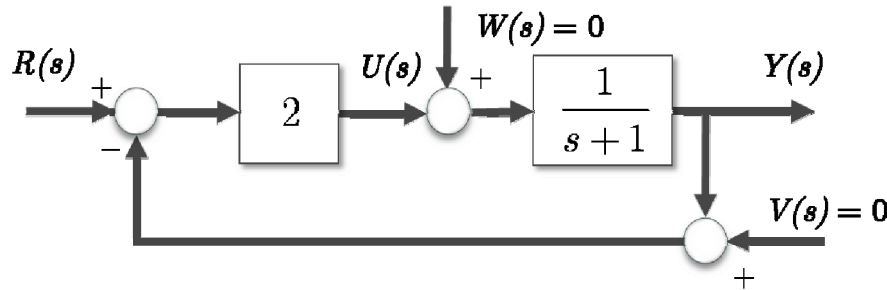
- Escalón de valor  $A$ :  $R(s) = \frac{A}{s}$

- Rampa de pendiente  $P$ :  $R(s) = \frac{P}{s^2}$

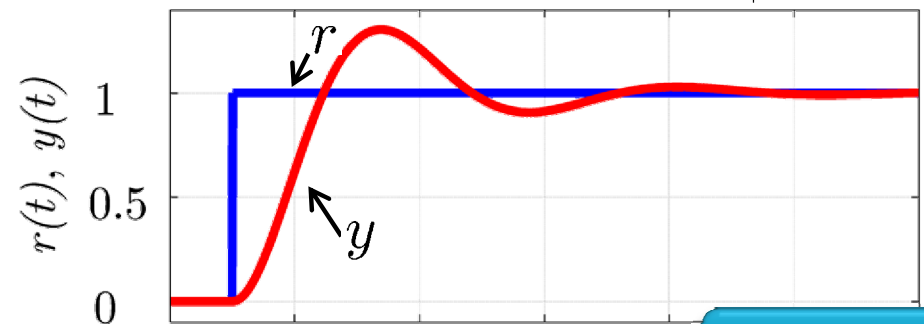
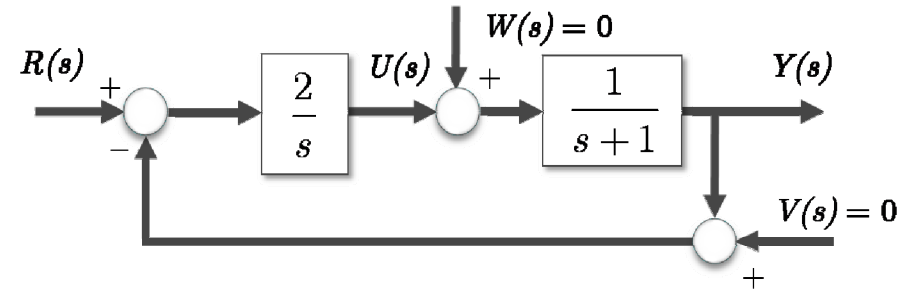
- Parábola no unitaria:  $R(s) = \frac{Q}{s^3}$



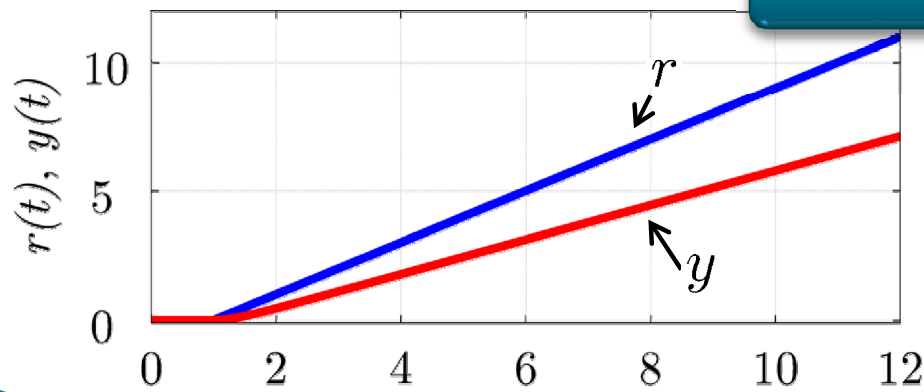
# Errores en régimen permanente



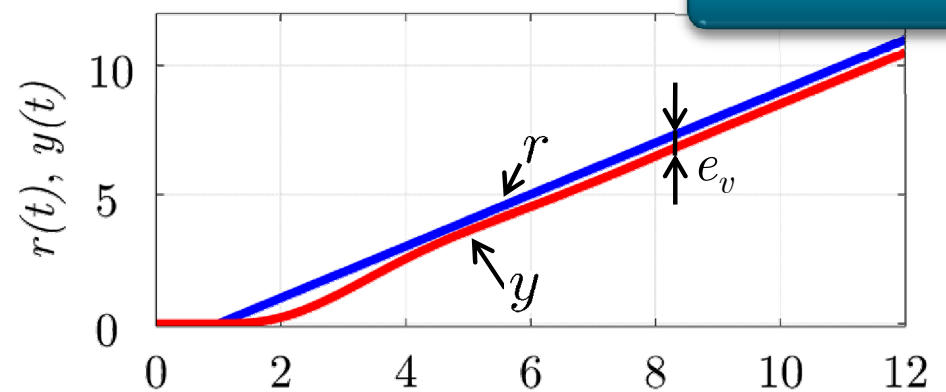
Tipo 0



Tipo 1

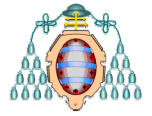


Tiempo (s)

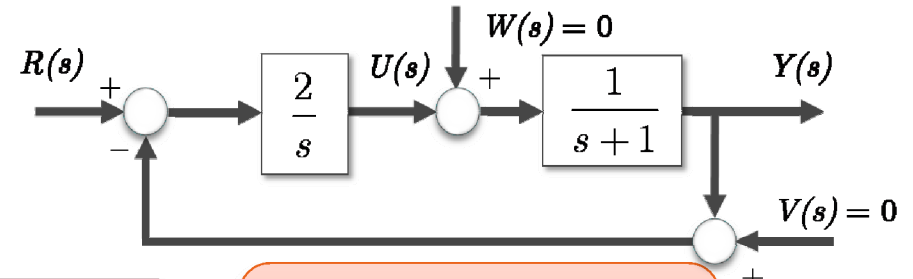
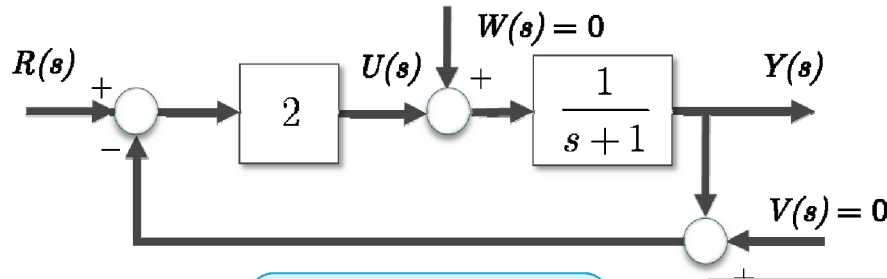


Tiempo (s)





# Errores en régimen permanente



$$S(s) = \frac{s+1}{s+3}$$

$$e_p = \lim_{s \rightarrow 0} s S(s) \frac{1}{s}$$

$$e_v = \lim_{s \rightarrow 0} s S(s) \frac{1}{s^2}$$

$$e_a = \lim_{s \rightarrow 0} s S(s) \frac{1}{s^3}$$

$$S(s) = \frac{s(s+1)}{s^2+s+2}$$

$$e_p = \lim_{s \rightarrow 0} s \frac{s+1}{s+3} \frac{1}{s} = \frac{1}{3}$$

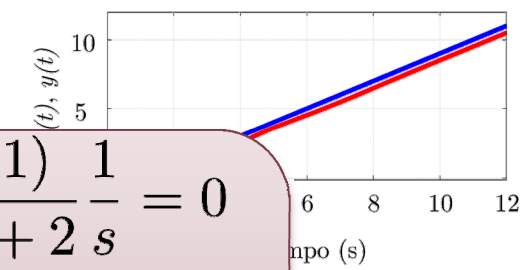
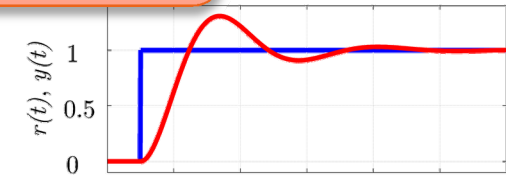
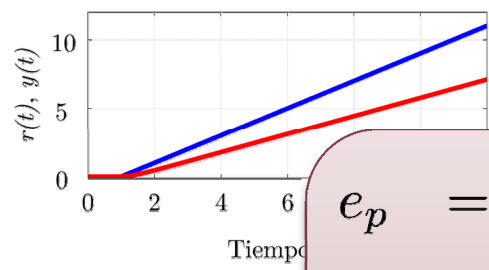
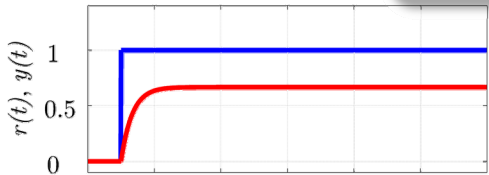
$$e_v = \lim_{s \rightarrow 0} s \frac{s+1}{s+3} \frac{1}{s^2} = \infty$$

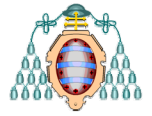
$$e_a = \lim_{s \rightarrow 0} s \frac{s+1}{s+3} \frac{1}{s^3} = \infty$$

$$e_p = \lim_{s \rightarrow 0} s \frac{s(s+1)}{s^2+s+2} \frac{1}{s} = 0$$

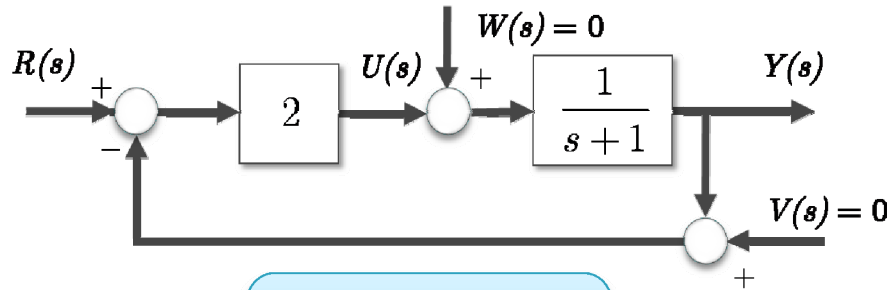
$$e_v = \lim_{s \rightarrow 0} s \frac{s(s+1)}{s^2+s+2} \frac{1}{s^2} = \frac{1}{2}$$

$$e_a = \lim_{s \rightarrow 0} s \frac{s(s+1)}{s^2+s+2} \frac{1}{s^3} = \infty$$



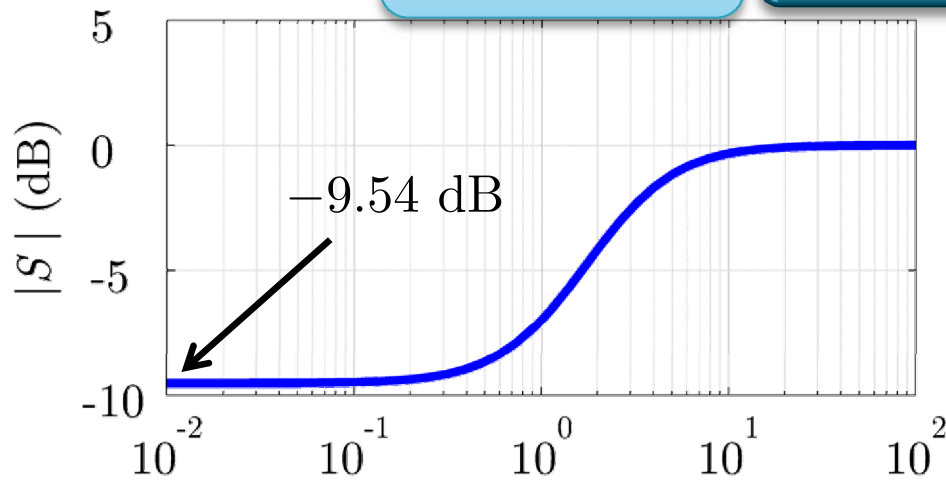


# Errores en régimen permanente



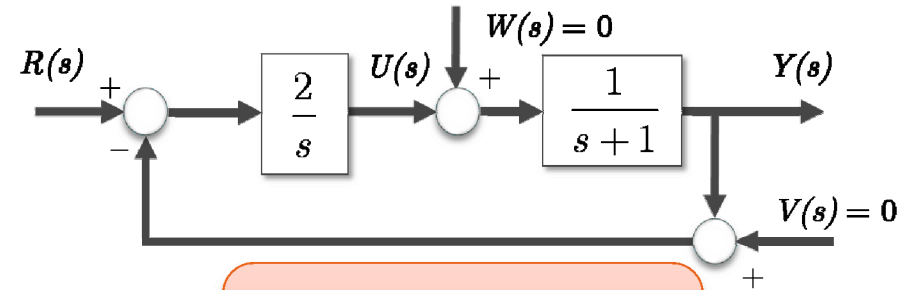
$$S(s) = \frac{s+1}{s+3}$$

Tipo 0



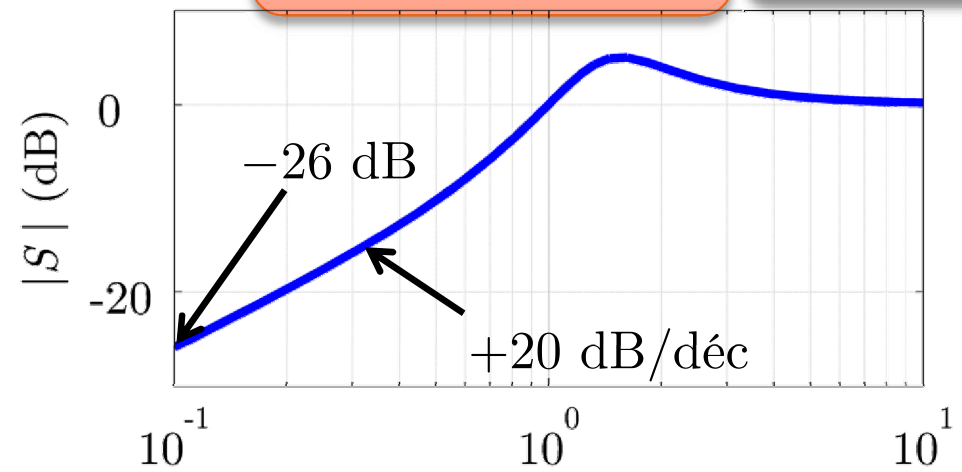
Frecuencia (rad/s)

$$-9.54 \text{ dB} \equiv 0.333$$



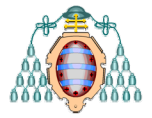
$$S(s) = \frac{s(s+1)}{s^2+s+2}$$

Tipo 1

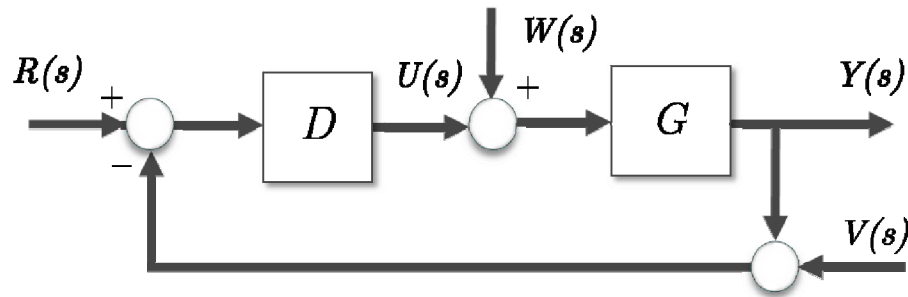


Frecuencia (rad/s)

$$\begin{aligned} -26 \text{ dB} &\equiv 0.0502 \\ .0502 / .1 &= .502 \end{aligned}$$



# Errores en régimen permanente



$$S = \frac{1}{1 + DG} = \frac{1}{1 + L}$$

Tipo 0

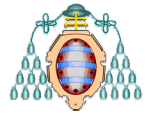
$$e_p = \lim_{s \rightarrow 0} s \frac{1}{1 + L(s)} \frac{1}{s} = \lim_{s \rightarrow 0} \frac{1}{1 + L(s)} = \frac{1}{1 + K_p} \quad K_p = \lim_{s \rightarrow 0} L(s)$$

Tipo 1

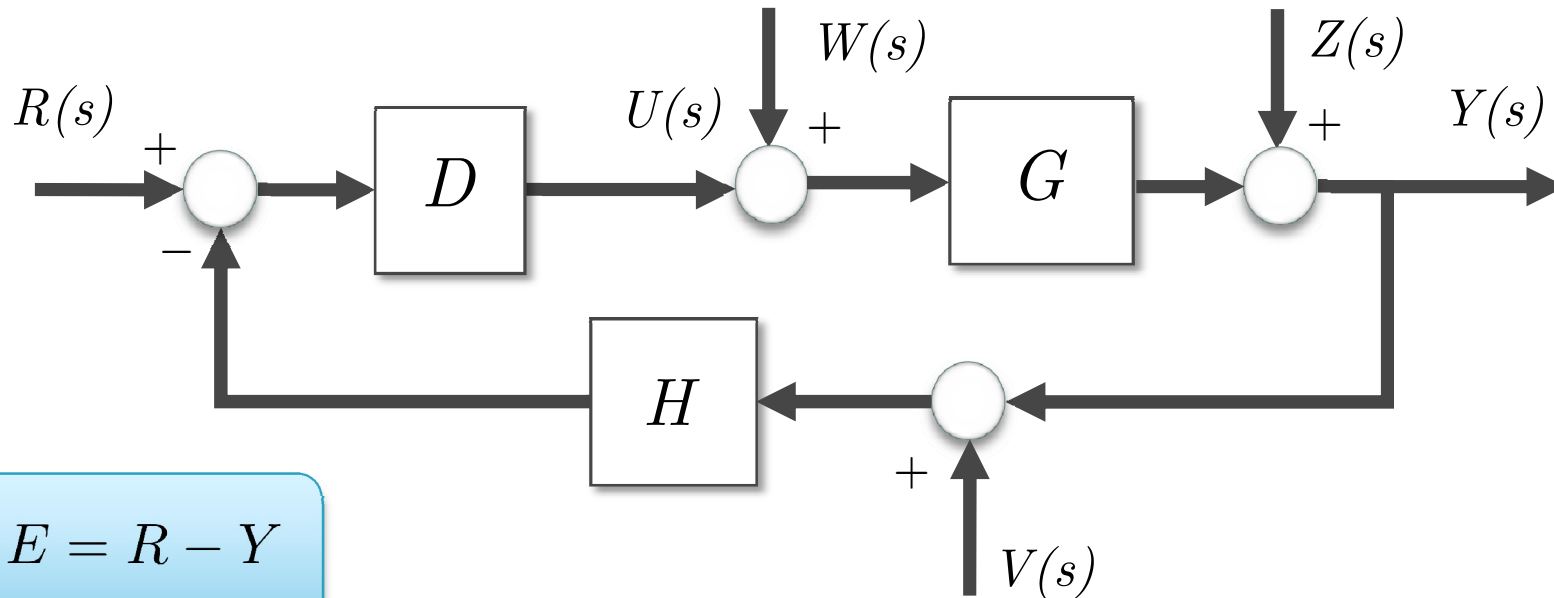
$$e_v = \lim_{s \rightarrow 0} s \frac{1}{1 + L(s)} \frac{1}{s^2} = \lim_{s \rightarrow 0} \frac{1}{sL(s)} = \frac{1}{K_v} \quad K_v = \lim_{s \rightarrow 0} sL(s)$$

Tipo 2

$$e_a = \lim_{s \rightarrow 0} s \frac{1}{1 + L(s)} \frac{1}{s^3} = \lim_{s \rightarrow 0} \frac{1}{s^2 L(s)} = \frac{1}{K_a} \quad K_a = \lim_{s \rightarrow 0} s^2 L(s)$$



# Errores en régimen permanente

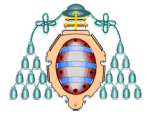


$$E = R - Y$$

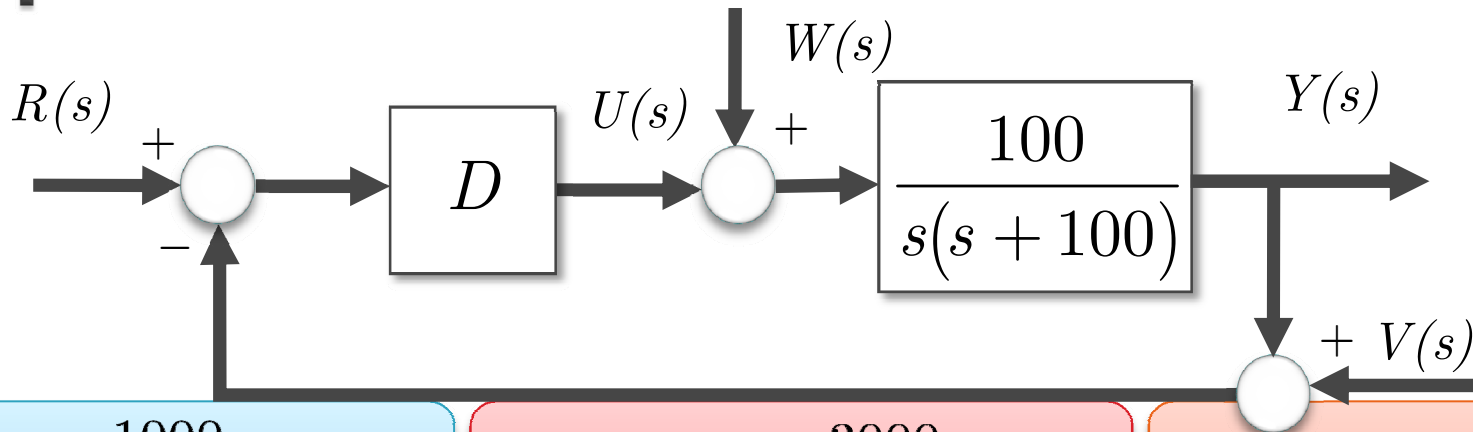
$$Y = \frac{DG}{1 + DGH} R$$

$$E = \frac{1 + DGH - DG}{1 + DGH} R$$

$$e_{rp} = \lim_{s \rightarrow 0} s \frac{1 + DGH(s) - DG(s)}{1 + DGH(s)} R(s)$$



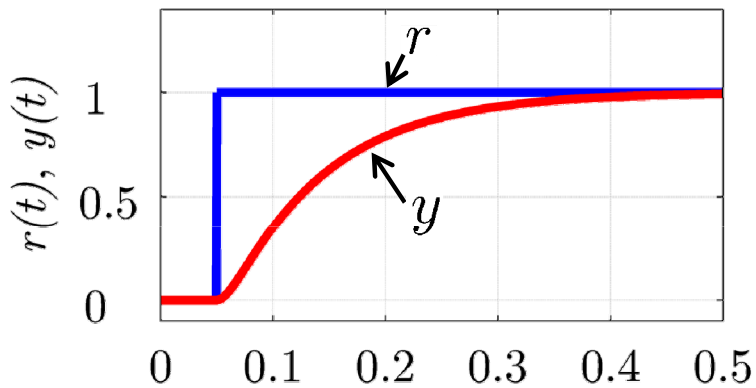
# Respuesta dinámica



$$T(s) = \frac{1000}{(s + 88.73)(s + 11.27)}$$

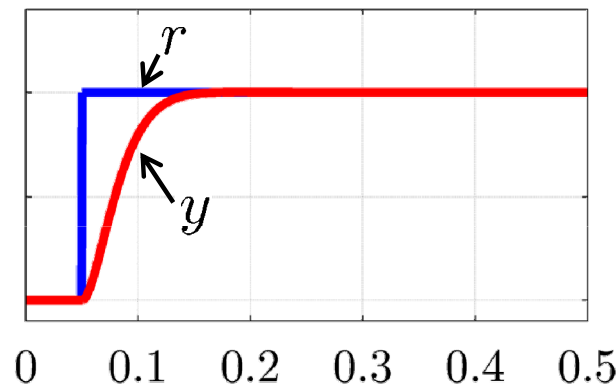
$$T(s) = \frac{3000}{s^2 + 100s + 3000}$$

$$T(s) = \frac{10000}{s^2 + 100s + 10000}$$



Tiempo (s)

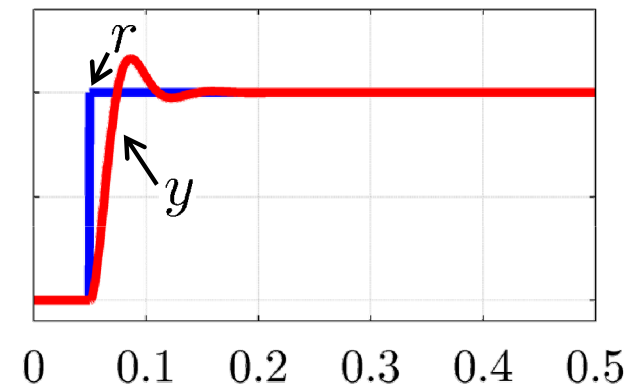
$$D = 10$$



Tiempo (s)

$$D = 30$$

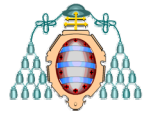
$$s_{1,2} = -50 \pm 10\sqrt{5}j$$



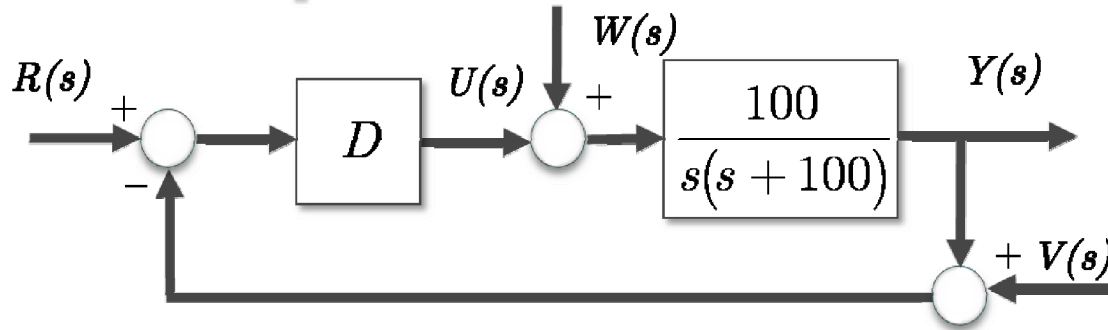
Tiempo (s)

$$D = 100$$

$$s_{1,2} = -50 \pm 50\sqrt{3}j$$



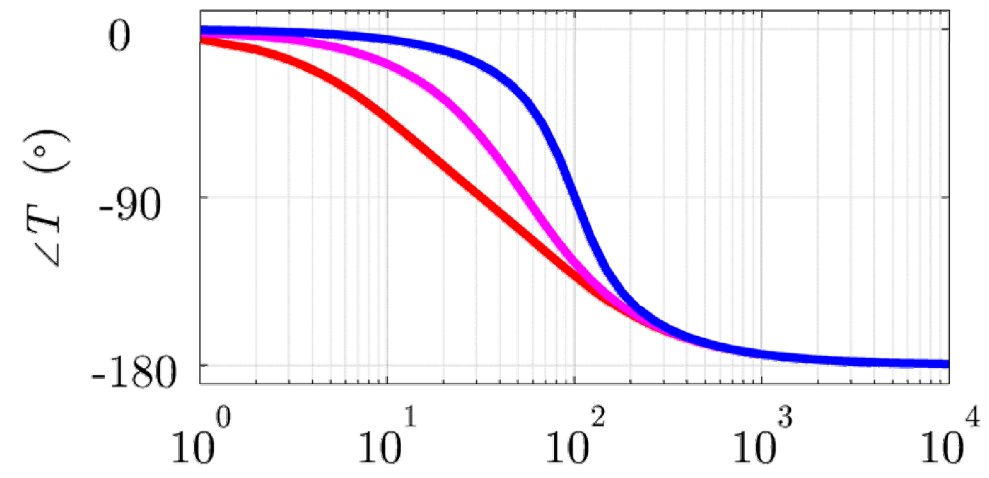
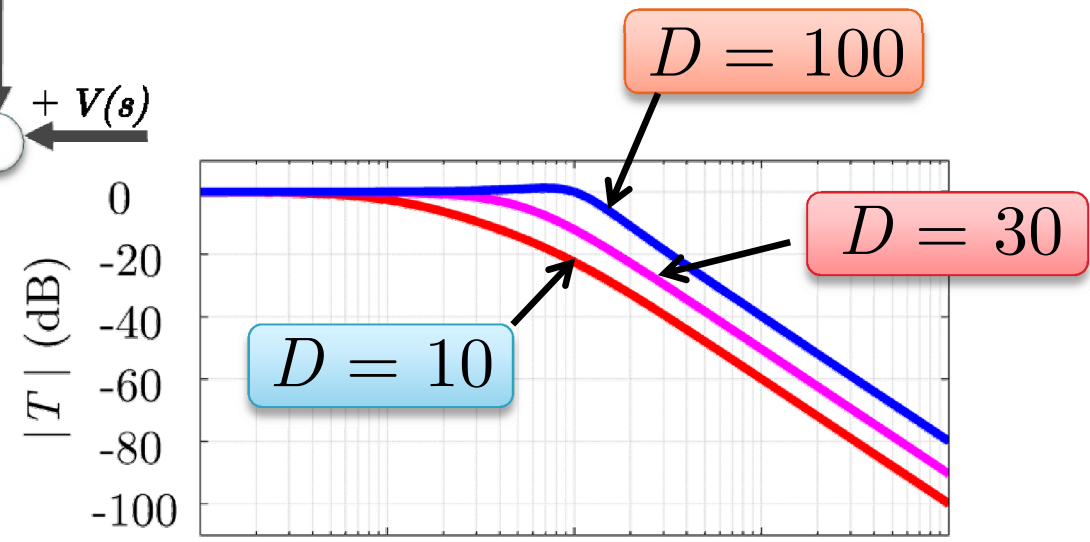
# Respuesta dinámica



$$T(s) = \frac{1000}{(s + 88.73)(s + 11.27)}$$

$$T(s) = \frac{3000}{s^2 + 100s + 3000}$$

$$T(s) = \frac{10000}{s^2 + 100s + 10000}$$



Frecuencia (rad/s)